

Mass-Generation by Weyl-Symmetry Breaking

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Abstract

A massless electroweak theory for leptons is formulated in a Weyl space, W_4 , yielding a Weyl invariant dynamics of a scalar field ϕ , chiral Dirac fermion fields ψ_L and ψ_R , and the gauge fields $\kappa_\mu, A_\mu, Z_\mu, W_\mu$ and W_μ^\dagger allowing for conformal rescalings of the metric $g_{\mu\nu}$ and all fields with nonvanishing Weyl weight together with the corresponding transformations of the Weyl vector fields, κ_μ , representing the $D(1)$ or dilatation gauge fields. The local group structure of this Weyl-electroweak (WEW) theory is given by $G = SO(3,1) \otimes D(1) \otimes \tilde{G}$ — or its universal covering group \bar{G} for the fermions — with \tilde{G} denoting the electroweak gauge group $SU(2)_W \times U(1)_Y$. In order to investigate the appearance of nonzero masses in the theory the Weyl-symmetry is explicitly broken by a term in the Lagrangean constructed with the curvature scalar R of the W_4 and a mass term for the scalar field. Thereby also the Z_μ - and W_μ - gauge fields as well as the charged fermion field (electron) acquire a mass as in the standard electroweak theory. The symmetry breaking is governed by the relation $D_\mu \Phi^2 = 0$, where Φ is the modulus of the scalar field and D_μ denotes the Weyl-covariant derivative. This true symmetry reduction, establishing a scale of length in the theory by breaking the $D(1)$ gauge symmetry, is compared to the so-called spontaneous symmetry breaking in the standard electroweak theory which is, actually, the choice of a particular (nonlinear) gauge obtained by adopting an origin, $\hat{\phi}$, in the coset space representing ϕ with $\hat{\phi}$ being invariant under the electromagnetic gauge group $U(1)_{e.m.}$. Particular attention is devoted to the appearance of Einstein's equations for the metric after the Weyl-symmetry breaking yielding a pseudo-Riemannian space, V_4 , from a W_4 and a scalar field with a constant modulus $\hat{\phi}_0$. The quantity $\hat{\phi}_0^2$ affects Einstein's gravitational constant in a manner comparable to the Brans-Dicke theory. The consequences of the broken WEW theory are worked out and the determination of the parameters of the theory is discussed.

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I. INTRODUCTION

The standard model in particle physics was very successful in giving a unified description of strong, weak and electromagnetic processes. However, it was always considered a problem by many theorists to understand properly the *mass giving algorithm* in this theory, i.e. to answer the question what lies behind the so-called “Higgs phenomenon”. Is this so-called spontaneous symmetry breaking by the vacuum expectation value of a scalar quantum field — yielding at the same time the gauge boson and the fermion masses in the theory — the correct way to account for the appearance of nonzero masses in physics? In other words, is the Higgs mechanism of the standard model only a convenient algorithm to generate masses of the gauge and fermion fields without loosing the renormalizability of the theory, or has this enigmatic phenomenon a deeper understanding with the present formulation of the theory yielding only a *particular parametrization* of a so far not very well understood aspect of the theoretical description: the origin of nonzero masses in nature? Related to this is the question whether the scalar quantum field ϕ needed to formulate the Higgs mechanism is, indeed, a true spin zero matter field which materializes as a spinless particle of a certain mass showing up in high energy processes.

Recently there have appeared several suggestions of a different nature and interpretation for the mass-giving mechanism in particle physics which would not require a Higgs particle to exist. The proposal of Pawłowski and Rączka (see [1] and references therein) stresses the conformal invariance of the original theory without actually breaking the conformal symmetry in the course of introducing masses, but rather considering the appearance of nonzero mass ratios as a certain choice of gauge. The other method which was proposed by the present author in collaboration with H. Tann [2] aims at accounting for the origin of nonzero masses by using a Weyl-geometric framework starting from an originally massless Weyl-symmetric theory and a subsequent explicit breaking of the Weyl symmetry with the help of a term in the Lagrangean involving the curvature scalar R of the Weyl space W_4 and the mass of the scalar field. In this framework the mass generation in an originally massless and scaleless theory is considered to be due to an interplay between the ambient geometry — a Weyl-geometry — and a universal scalar quantum field. The breaking $W_4 \rightarrow V_4$ yielding a pseudo-Riemannian, i.e. V_4 , description in the limit is formulated as a condition on the Weyl vector fields κ_ρ which in turn represent an aspect of the Weyl-geometry (see Sect. II below) being given in the broken case as a derivative of the scalar field yielding thus, finally, zero length curvature, i.e. $f_{\mu\nu} = 0$. [The Weyl vector fields κ_ρ are the $D(1)$ or dilatation gauge potentials and $f_{\mu\nu}$ are the corresponding curvature components.] At the same time the validity of Einstein’s equations for the metric in the V_4 limit is required to be satisfied with the energy-momentum tensors of the now massive fields as sources and with — as it turns out — a gravitational coupling constant depending on the modulus Φ of ϕ . On the one hand, i.e. as far as gravitation is concerned, the squared modulus, Φ^2 , of the scalar field plays the role of a Brans-Dicke-type field in this formalism [3]. On the other hand, i.e. as far as the generation of nonzero masses for the gauge and fermion fields is concerned, the field ϕ plays a role of a Higgs-type field in this broken Weyl theory.

The idea of our geometrically motivated method basically is that in accounting for the origin of nonzero masses in nature the theoretical framework should include gravitation from the outset starting thus from the investigation of a dynamics of massless boson and fermion

fields formulated in a Weyl space W_4 containing the dynamics of a metric modulo conformal rescalings. Subsequently the Weyl-symmetry is broken explicitly yielding a Riemannian description together with a definite length (and mass) scale being established and a set of field equations for the metric being required to be satisfied.

The plan of the paper is as follows. After some introductory remarks on Weyl geometry and Weyl spaces we briefly review the theory presented in [2] in which the geometry of a Weyl space of dimension four and its use in elementary particle theory was studied in detail. In this work the scalar field — called φ there — was a complex quantum field with nonzero Weyl weight carrying besides its transformation character under Weyl transformations [see below] no further representation properties. This Weyl covariant theory is then generalized in Sect. II by including the electroweak gauge group $\tilde{G} = SU(2)_W \times U(1)_Y$ in the description with the scalar field — now denoted by ϕ — possessing in addition representation properties with respect to the weak isospin group $SU(2)_W$ as well as the hypercharge group $U(1)_Y$. After some general remarks about symmetry breaking in gauge theories at the end of Sect. II, this Weyl-electroweak theory (WEW theory) is explicitly broken in Sect. III and the role of electromagnetism (Subsection A) and gravitation (Subsection B) in the resulting theory, formulated in a Riemannian space, is investigated in detail. Subsection C deals with the wave equation for the scalar field ϕ , and Subsection D, finally, is devoted to the determination of the free parameters of the theory. An essential point in the presented discussion is that the breaking of the Weyl-symmetry and the appearance of a length and mass scale in the theory is qualitatively different from the so-called spontaneous symmetry breaking in the electroweak theory yielding the masses of the gauge boson and fermion fields there. We show, using the coset representation of the scalar field ϕ derived and discussed in Appendix A and B, that the so-called spontaneous symmetry breaking is a particular choice of gauge within the electroweak theory which is characterized by the choice of an origin $\hat{\phi}$ in the coset space, with $\hat{\phi}$ being invariant under the *electromagnetic* gauge group $U(1)_{e.m.}$ exhibiting thus, finally, a residual $U(1)_{e.m.}$ gauge symmetry of the theory which is, in fact, a nonlinear realization of the original \tilde{G} -symmetry on the subgroup $U(1)_{e.m.}$. We end in Sect. IV with some concluding remarks and discussions of the results obtained due to the true symmetry breaking occurring in this Weyl-electroweak theory when a definite intrinsic unit of lengths is established.

II. ELECTROWEAK THEORY IN WEYL-SYMMETRIC FORM

A. Geometric Preliminaries.

In this section we investigate a unified electroweak theory in the presence of “gravitation” formulated in a Weyl space. Since Einstein’s metric theory of gravitation is neither conformally invariant nor a theory which could be formulated in a Weyl space we have to break the Weyl-symmetry in a second step, as mentioned, in order to recover Einstein’s theory in this framework. On the other hand, we want to generate masses for the matter fields of the theory by starting from a gauge dynamics involving at first only massless fields in order to see how this mass generation obtained by Weyl-symmetry breaking compares to the Higgs mechanism in the standard model. To facilitate our discussion we shall, however, disregard the $SU(3)$ gauge group of colour and shall treat only the electroweak part of the

standard model together with gravitation neglecting thus the strong interactions. Moreover, we treat only one generation of fermions for simplicity, i.e. the leptons e and $\nu = \nu_e$. Our main interest here is to see how the electron mass m_e and the $SU(2)$ -gauge boson masses appear due to Weyl-symmetry breaking. The fact that a second fermion generation with $m_\mu > m_e$ introduces *another* and *different* mass scale given by the myon mass m_μ — opening up, moreover, the possibility of μ_{e3} -decay — and, similarly, for the third generation with a tau-mass $m_\tau > m_\mu$, cannot be explained by the present model. In this respect the broken Weyl theory is, unfortunately, not better than the standard model.

In order to stress the role of gravitation in the present context, we like to make the following remarks. The usual Higgs mechanism yielding nonzero masses for the fields in the standard model does not limit the actual size of the masses obtained: The mass value for the fermion fields could be shifted to arbitrary large values. Of course, the gauge boson masses are related to the strength of the Fermi coupling constant for charge changing weak currents. However, the feature that the elementary fermion masses could be arbitrary large seems to be an unphysical one since the generation of nonzero rest masses is accompanied by the generation of gravitational fields and gravitational interactions. If gravity were included in the standard model one would expect in a consistent theory that elementary masses cannot be shifted to arbitrary large values due to damping effects resulting from the consideration of gravitational interactions. This is reminiscent of Hermann Weyl's remark that a theory which tries to account for the origin of masses in nature cannot be formulated consistently without considering gravitation at the same time. The standard model should thus be formulated in a general relativistic setting. In starting from a massless conformally invariant scenario a formulation of the original dynamics in a Weyl space or Weyl geometry would thus be very suggestive. Let us, therefore, begin our investigation by formulating a gauge dynamics of a massless spin zero and a single generation of massless spin $\frac{1}{2}$ fields in a Weyl space W_4 .

In [2] the geometry of a Weyl space was investigated in detail. We shall use the notation defined there (see in particular Appendix A of this paper). We shall refer to [2] as to I in the following and refer, for example, to Eq.(1.2) of I as to (I, 1.2) etc..

A Weyl space W_4 is characterized by two differential forms:

$$ds^2 = g_{\mu\nu}(x)dx^\mu \otimes dx^\nu; \quad \kappa = \kappa_\mu(x)dx^\mu. \quad (2.1)$$

A W_4 is equivalent to a family of Riemannian spaces

$$(g_{\mu\nu}, \kappa_\sigma), (g'_{\mu\nu}, \kappa'_\sigma), (g''_{\mu\nu}, \kappa''_\sigma) \dots \quad (2.2)$$

with metrics $g_{\mu\nu}(x)$, $g'_{\mu\nu}(x) \dots$ and Weyl vector fields $\kappa_\rho(x)$, $\kappa'_\rho(x) \dots$ related by

$$g'_{\mu\nu}(x) = \sigma(x) g_{\mu\nu}(x) \quad (2.3)$$

$$\kappa'_\rho(x) = \kappa_\rho(x) + \partial_\rho \log \sigma(x), \quad (2.4)$$

where $\sigma(x) \in D(1)$, $\sigma(x) = e^{\rho(x)} > 0$, with $D(1)$ denoting the dilatation group which is isomorphic to R^+ (the positive real line). The transformations (2.3) and (2.4) are called *Weyl-transformations* involving a conformal rescaling (2.3) of the metric together with the transformation (2.4) of the Weyl vector fields. In the following discussion we shall consider a Weyl space of dimension $d = 4$ possessing Lorentzian signature $(+, -, -, -)$ of its metrics.

(For reference to the earlier history of Weyl spaces and Weyl geometry see the references quoted in I.)

A W_4 reduces to a Riemannian space V_4 for $\kappa_\mu = 0$; a W_4 is equivalent to a V_4 if the “length curvature” associated with the Weyl vector field κ_μ is zero, i.e. for

$$f_{\mu\nu} = \partial_\mu \kappa_\nu - \partial_\nu \kappa_\mu = 0. \quad (2.5)$$

In I we studied a Weyl invariant dynamics of massless fields involving the metric $g_{\mu\nu}$, the Weyl vector or $D(1)$ gauge fields κ_ρ , and the electromagnetic, i.e. $U(1) = U(1)_{e.m.}$ gauge fields A_μ , as well as the “matter” fields φ (spin zero) and ψ (spin $\frac{1}{2}$, Dirac spinor) with Weyl weight $w(\varphi) = -\frac{1}{2}$ and $w(\psi) = -\frac{3}{4}$ [see I]. It turned out in the discussion given in I that φ is not a bona fide matter field but could better be characterized as a universal Bans-Dicke-type field or a Higgs-type field related to symmetry breaking. On the other hand, the spinor field ψ is a true matter field representing leptons in the present formalism.

The local group structure of the theory studied in I was $SO(3,1) \otimes D(1) \otimes U(1)$ for the nonfermionic fields and $Spin(3,1) \otimes D(1) \otimes U(1)$ for the Dirac spinor field ψ with $Spin(3,1)$ denoting the universal covering group of the orthochronous Lorentz group $SO(3,1) \equiv O(3,1)^{++}$ acting in the local spin space \mathbf{C}_4 representing the standard fiber of the spinor bundle on which the field $\psi(x)$ is defined as a section (see I and the discussion below), and with $U(1)$ denoting the electromagnetic gauge group.

The pull back of a connection on the corresponding frame bundle was denoted by the one-forms of Weyl weight zero (Latin indices are local Lorentzian indices):

$$(w_{ik} = -w_{ki}, \kappa, A) \quad (2.6)$$

with coefficients with respect to a natural base dx^μ in the dual tangent space $T_x^*(W_4)$ to W_4 at x given by:

$$(\Gamma_{\mu ik}(x) = -\Gamma_{\mu ki}(x), \kappa_\mu(x), A_\mu(x)). \quad (2.7)$$

The first entry in (2.6) is Lorentz-valued (i.e. antisymmetric in i and k), the second is $D(1)$ -valued (corresponding to a real, noncompact, abelian gauge group), and the third is $U(1)$ -valued (corresponding to the complex, compact, abelian electromagnetic gauge group). The fully covariant derivative of a tensor quantity $\phi^{(n,m)}$ of type (n,m) , i.e. covariant of degree n and contravariant of degree m , possessing Weyl weight $w(\phi^{(n,m)})$ and charge q , i.e. transforming under Weyl transformations (2.3) and (2.4) as

$$\phi^{(n,m)'}(x) = [\sigma(x)]^{w(\phi^{(n,m)})} \phi^{(n,m)}(x), \quad (2.8)$$

and under electromagnetic gauge transformations as

$$\phi^{(n,m)'}(x) = e^{-\frac{iq}{\hbar c}} \phi^{(n,m)}(x), \quad (2.9)$$

is given by

$$\begin{aligned} \tilde{D}\phi^{(n,m)} &= D\phi^{(n,m)} + \frac{iq}{\hbar c} A\phi^{(n,m)} \\ &= \nabla\phi^{(n,m)} - \omega(\phi^{(n,m)})\kappa\phi^{(n,m)} + \frac{iq}{\hbar c} A\phi^{(n,m)}. \end{aligned} \quad (2.10)$$

Here $D = dx^\mu D_\mu$ denotes the Weyl-covariant derivative (I, A3), and $\kappa = \kappa_\mu dx^\mu$, $A = A_\mu dx^\mu$. Furthermore, $\nabla = dx^\mu \nabla_\mu$ with ∇_μ denoting the covariant derivative with respect to the Weyl-connection $\Gamma_{\mu\nu}^\rho$ defined by

$$\Gamma_{\mu\nu}^\rho = \bar{\Gamma}_{\mu\nu}^\rho + W_{\mu\nu}^\rho = \frac{1}{2}g^{\rho\lambda}(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}) - \frac{1}{2}(\kappa_\mu \delta_\nu^\rho + \kappa_\nu \delta_\mu^\rho - \kappa^\rho g_{\mu\nu}). \quad (2.11)$$

Here and in the sequel purely metric quantities pertaining to a V_4 are denoted by a bar, for example, $\bar{\Gamma}_{\mu\nu}^\rho = \{\bar{\Gamma}_{\mu\nu}^\rho\}$ are the Christoffel symbols of the metric $g_{\mu\nu}$. We remark in passing that the connection coefficient $\Gamma_{\mu\nu}^\rho$, defined with respect to a natural base in (2.11), is Weyl-invariant, obeying $\Gamma_{\mu\nu}^\rho = \Gamma_{\mu\nu}^{\rho'}$, with the change in the metric computed according to (2.3) being compensated by the change in the Weyl vector fields computed according to (2.4). Thus the Weyl-covariant derivative $D\phi^{(n,m)}$ of a quantity $\phi^{(n,m)}$ is independent of the Weyl gauge chosen in the family (2.2), and, by definition, transforms again like (2.8). Corresponding to Eq.(2.10) the Weyl- and $U(1)$ - covariant derivative of a spinor field with $w(\psi) = -\frac{3}{4}$ and charge e is given by

$$\begin{aligned} \tilde{D}\psi(x) &= D\psi(x) + \frac{ie}{\hbar c} A \cdot \mathbf{1} \psi(x) \\ &= dx^\mu \left\{ \left(\partial_\mu + i\Gamma_\mu(x) \right) \psi(x) + \frac{3}{4} \kappa \cdot \mathbf{1} \psi(x) + \frac{ie}{\hbar c} A_\mu \cdot \mathbf{1} \psi(x) \right\}, \end{aligned} \quad (2.12)$$

where $\Gamma_\mu(x)$ is the spin connection

$$\Gamma_\mu(x) = \lambda_\mu^j(x) \frac{1}{2} \Gamma_{jik}(x) S^{ik}; \quad S^{ik} = \frac{i}{4} [\gamma^i, \gamma^k]. \quad (2.13)$$

Here γ^i ; $i = 0, 1, 2, 3$ are the constant Dirac matrices satisfying $\{\gamma^i, \gamma^k\} = \gamma^i \gamma^k + \gamma^k \gamma^i = 2\eta^{ik} \cdot \mathbf{1}$ with $\eta^{ik} = \text{diag}(1, -1, -1, -1)$ and $\lambda_\mu^j(x)$ are the vierbein fields obeying

$$g_{\mu\nu}(x) = \lambda_\mu^i(x) \lambda_\nu^k(x) \eta_{ik}. \quad (2.14)$$

The inverse vierbein fields used below are denoted by $\lambda_i^\mu(x)$. The quantities $\Gamma_{\mu ik} = \lambda_\mu^j \Gamma_{jik}$ appearing in the first equation of (2.13) are the coefficients of the Lorentz part of the connection mentioned in relation to (2.6) and (2.7) with Γ_{jik} defined by

$$\omega_{ik} = \bar{\omega}_{ik} - \frac{1}{2}(\kappa_i \theta_k - \kappa_k \theta_i) = \theta^j \Gamma_{jik} \quad (2.15)$$

where $\bar{\omega}_{ik} = \theta^j \bar{\Gamma}_{jik}$ with $\bar{\Gamma}_{jik} = -\bar{\Gamma}_{jki}$ being the Ricci rotation coefficients of a V_4 and $-\frac{1}{2}(\kappa_i \eta_{kj} - \kappa_k \eta_{ij}) \theta^j$ denoting the Weyl addition in a W_4 . In Eq. (2.15) $\theta^j = \lambda_\mu^j(x) dx^\mu$; $j = 0, 1, 2, 3$ are the fundamental one-forms representing a Lorentzian basis in the dual tangent space, $T_x^*(W_4)$, at $x \in W_4$. (Compare Appendix A of I.) The form (2.15) for ω_{ik} together with (2.11) yields $D\lambda_i^\mu = 0$ with $w(\lambda_i^\mu) = -\frac{1}{2}$ for the Weyl-covariant derivative of the vierbein field and, correspondingly, $D_\rho g_{\mu\nu} = 0$ with the Weyl weights $w(g_{\mu\nu}) = 1$ [see Eq. (2.3)], $w(\lambda_\mu^i) = \frac{1}{2}$ and $w(\eta_{ik}) = 0$ [compare (2.14)]. The relation $D_\rho g_{\mu\nu} = 0$, expressing the fact that the metric is Weyl-covariant constant, reduces the connection on the general linear frame bundle [i.e. the $Gl(4, R)$ -bundle] in a Weyl space — possessing a metric given only modulo conformal transformations (2.3) — to a Weyl frame bundle, called P_W in I,

possessing the structural group $SO(3, 1) \otimes D(1)$.

B. Standard Model Extension: Weyl-Electroweak Theory (WEW Theory)

We now extend the formalism developed in I to a unified electroweak theory [neglecting as mentioned $SU(3)$ colour degrees of freedom] by extending the gauge group of the theory to the group [4]

$$G = SO(3, 1) \otimes D(1) \otimes U(1)_Y \times SU(2)_W, \quad (2.16)$$

i.e. interpreting the $U(1)$ degree of freedom in I as weak hypercharge, $U(1)_Y$, and considering an additional weak isospin group $SU(2)_W$ (compare Weinberg's model of leptons [5]). The underlying principal bundle over W_4 is now

$$P = P(W_4, G) \quad (2.17)$$

with G given by (2.16). For the discussion of spinor fields of Dirac type one considers, as usual, the spin frame bundle $\bar{P} = \bar{P}(W_4, \bar{G})$ possessing the universal covering group of G , i.e.

$$\bar{G} = Spin(3, 1) \otimes D(1) \otimes U(1)_Y \times SU(2)_W \quad (2.18)$$

as structural group (compare the discussion above and in I). A Dirac spinor field ψ , with Weyl weight $w(\psi) = -\frac{3}{4}$ as before [see I], and hypercharge Y , possessing a definite representation character regarding $SU(2)_W$ i.e. [we follow the standard model assignment] $I = \frac{1}{2}$ (isodoublet), $Y = -\frac{1}{2}$ for the left-handed chiral fields $\psi_L(x) = \frac{1}{2}(1 - \gamma_5)\psi(x)$, and $I = 0$ (isosinglet), $Y = -1$, for the right-handed chiral fields $\psi_R(x) = \frac{1}{2}(1 + \gamma_5)\psi(x)$, with

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3; \quad \gamma_5^\dagger = \gamma_5; \quad (\gamma_5)^2 = 1. \quad (2.19)$$

The chiral fields $\psi_L(x)$ and $\psi_R(x)$ will be regarded, respectively, as a section on the spinor bundle S associated to \bar{P} with fiber F given by $\tilde{\mathbf{C}} = \mathbf{C}_4 \times \mathbf{C}_2$ for $I = \frac{1}{2}$; or given by $\tilde{\mathbf{C}} = \mathbf{C}_4 \times \mathbf{C}$ for $I = 0$, being thus defined by the bundle

$$S = S(W_4, F = \tilde{\mathbf{C}}, \bar{G}). \quad (2.20)$$

Hence the leptonic chiral fermion fields of Weyl weight $-\frac{3}{4}$ will be

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(1 - \gamma_5)\psi_\nu \\ \frac{1}{2}(1 - \gamma_5)\psi_e \end{pmatrix}, Y = -\frac{1}{2}; \quad \psi_R = e_R = \frac{1}{2}(1 + \gamma_5)\psi_e, Y = -1, \quad (2.21)$$

with their adjoints ($\bar{\psi} = \psi^\dagger \gamma_0$):

$$\bar{\psi}_L = (\bar{\nu}_L, \bar{e}_L) = \left(\bar{\psi}_\nu \frac{1}{2}(1 + \gamma_5), \bar{\psi}_e \frac{1}{2}(1 + \gamma_5) \right), Y = \frac{1}{2}; \quad \bar{\psi}_R = \bar{e}_R = \bar{\psi}_e \frac{1}{2}(1 - \gamma_5); Y = 1. \quad (2.22)$$

For the scalar field we shall use as representation character with respect to $SU(2)_W$ an isodoublet, $I = \frac{1}{2}$, yielding thus

$$\phi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix} \quad \text{with} \quad Y = \frac{1}{2}; \quad \text{and} \quad \phi^\dagger = (\varphi_+^*, \varphi_0^*) \quad \text{with} \quad Y = -\frac{1}{2} \quad (2.23)$$

possessing the Weyl weight $w(\phi) = w(\phi^\dagger) = -\frac{1}{2}$. Here φ_0 is a neutral complex field, and φ_+ is a complex field with positive charge, obeying $\varphi_+^* = \varphi_-$. The relation between electric charge Q , isospin (I_3), and weak hypercharge is, as usual,

$$Q = I_3 + Y. \quad (2.24)$$

The field ϕ may be regarded as a section on the bundle

$$E = E(W_4, F = \mathbb{C}_2, G) \quad (2.25)$$

associated to P . The square of the modulus of the scalar field is now given by the $U(1)_Y$ and $SU(2)_W$ invariant of Weyl weight $w(\Phi^2) = -1$:

$$\Phi^2 = \phi^\dagger \phi = \varphi_+^* \varphi_+ + \varphi_0^* \varphi_0 = |\varphi_+|^2 + |\varphi_0|^2. \quad (2.26)$$

The invariant Yukawa coupling term of Weyl weight -1 for the scalar and spinor fields — reading $\tilde{\gamma} \sqrt{\varphi^* \varphi} (\bar{\psi} \psi) = \tilde{\gamma} \Phi (\bar{\psi} \psi)$ in I — will now be written as

$$\tilde{\gamma} \{ (\bar{\psi}_L \phi) \psi_R + \bar{\psi}_R (\phi^\dagger \psi_L) \}. \quad (2.27)$$

Calling the $U(1)_Y$ gauge potentials B_μ , i.e. reserving as usual the notation A_μ for the electromagnetic gauge potentials, the full G -covariant derivative of ϕ is written as [compare (2.16)]

$$\tilde{D}_\mu \phi = D_\mu \phi + \frac{i}{2} \tilde{g} A_\mu^a \tau_a \phi + i \tilde{g}' Y B_\mu \cdot \mathbf{1} \phi. \quad (2.28)$$

Here the Weyl-covariant part is given by $D_\mu \phi = \partial_\mu \phi + \frac{1}{2} \kappa_\mu \cdot \mathbf{1} \phi$, and the $SU(2)_W$ -gauge fields are denoted by A_μ^a ; $a = 1, 2, 3$; $\mu = 0, 1, 2, 3$. Moreover, $Y = \frac{1}{2}$ in (2.28) according to (2.23). The Lie algebra of $SU(2)_W$ adapted to the choice (2.23) for ϕ is given by $\frac{1}{2} \tau_a$ with τ_a ; $a = 1, 2, 3$ denoting the Pauli matrices [summation over a from 1 to 3 is understood in (2.28)]. Finally, \tilde{g} and \tilde{g}' are dimensionless coupling constants for the $SU(2)_W$ and $U(1)_Y$ coupling, respectively, which we write with a tilde in order not to confuse them with the determinant of the metric tensor called g .

A similar expression as (2.28) may be written down for $\tilde{D}_\mu \psi_L$ involving the Weyl-covariant part $D_\mu \psi_L = (\partial_\mu + i \Gamma_\mu) \psi_L + \frac{3}{4} \kappa_\mu \cdot \mathbf{1} \psi_L$, an $SU(2)_W$ part as in (2.28), and an $U(1)_Y$ part with $Y = -\frac{1}{2}$. [Compare (2.12).] For $\tilde{D}_\mu \psi_R$ the A_μ^a -contributions are absent due to the choice $I = 0$ for the right-handed fermion field. We take account of this in the notation by writing only one tilde for the covariant differentiation of ψ_R , i.e. $\tilde{D}_\mu \psi_R \equiv \tilde{D}_\mu \psi_R = D_\mu \psi_R + i \tilde{g}' Y B_\mu \cdot \mathbf{1} \psi_R$ with $Y = -1$ according to (2.21).

We are now in a position to write down a G -gauge invariant Lagrangean density of Weyl weight zero generalizing $\tilde{\mathcal{L}}_{W_4}$ of I [compare (I, 3.8)] to the case of an electroweak theory including “gravitation”, i.e. containing also a dynamics for the metric (determined modulo Weyl-transformations), in a scenario for the massless fields $\phi, \psi_L, \psi_R, \kappa_\mu, B_\mu$ and

A_μ^a possessing all a definit Weyl weight which is zero for the gauge fields $\kappa_\mu, B_\mu, A_\mu^a$, and is $w(\phi) = -\frac{1}{2}$ and $w(\psi_L) = w(\psi_R) = -\frac{3}{4}$ as mentioned above. Again we use below the same coefficient for the kinetic term of the scalar field and for the $\frac{1}{12}R$ -term implying in I the validity of the relation (I, 2.20), while (I, 2.21) is a consequence of the choice $w(\psi) = -\frac{3}{4}$. The Lagrangean for a massless G -invariant theory, called for short WEW theory (Weyl-electroweak theory), now reads:

$$\begin{aligned} \tilde{\mathcal{L}}_{W_4} = & K\sqrt{-g}\left\{\frac{1}{2}g^{\mu\nu}(\overrightarrow{\tilde{D}}_\mu\phi)^\dagger\overrightarrow{\tilde{D}}_\nu\phi - \frac{1}{12}R\phi^\dagger\phi - \beta(\phi^\dagger\phi)^2 + \tilde{\alpha}R^2\right. \\ & + \frac{i}{2}\left(\bar{\psi}_L\gamma^\mu\overrightarrow{\tilde{D}}_\mu\psi_L - \bar{\psi}_L\overleftarrow{\tilde{D}}_\mu\gamma^\mu\psi_L\right) + \frac{i}{2}\left(\bar{\psi}_R\gamma^\mu\overrightarrow{\tilde{D}}_\mu\psi_R - \bar{\psi}_R\overleftarrow{\tilde{D}}_\mu\gamma^\mu\psi_R\right) \\ & \left. + \tilde{\gamma}[(\bar{\psi}_L\phi)\psi_R + \bar{\psi}_R(\phi^\dagger\psi_L)] - \tilde{\delta}\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \tilde{\delta}\frac{1}{4}\left(F_{\mu\nu}^aF_a^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}\right)\right\}, \end{aligned} \quad (2.29)$$

Here γ^μ denote a set of x -dependent γ -matrices with Weyl weight $w(\gamma^\mu) = -\frac{1}{2}$ defined by

$$\gamma^\mu = \gamma^\mu(x) = \lambda_i^\mu(x)\gamma^i \quad \text{obeying} \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \cdot \mathbf{1}. \quad (2.30)$$

The meaning of the various terms in (2.29) is the same as in I and was described there in detail. This is true except for the generalization of the covariant derivatives, as explained above, being denoted here by $\overrightarrow{\tilde{D}}_\mu$ and $\overleftarrow{\tilde{D}}_\mu$ with the arrows \rightarrow and \leftarrow indicating, as usual, the action on ψ_L, ψ_R and on $\bar{\psi}_L, \bar{\psi}_R$, respectively, with a sign change involved according to the rule [compare (2.12) and (2.28)], $\overrightarrow{\tilde{D}}_\mu^\dagger = \gamma^0\overleftarrow{\tilde{D}}_\mu\gamma^0$ for the fermion fields and similarly for ϕ , i.e. $(\overrightarrow{\tilde{D}}_\mu\phi)^\dagger = \phi^\dagger\overleftarrow{\tilde{D}}_\mu$ with $\overleftarrow{\tilde{D}}_\mu = \overrightarrow{\tilde{D}}_\mu^\dagger = \tilde{D}_\mu^\dagger$. Corresponding to this, i.e. to the introduction of the new gauge fields B_μ and A_μ^a , the last two terms in (2.29) replace the electromagnetic term $-\frac{1}{K}\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ in (I, 3.8) with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Since the fields B_μ and A_μ^a have the dimension $[L^{-1}]$ (L =Length) as seen from (2.28), we replace the factor $\frac{1}{K}$ (see below) appearing in front of the electromagnetic term in (I, 3.8) by a constant $\tilde{\delta}$ of dimension $[L^2]$. Moreover, the Lagrangean $\tilde{\mathcal{L}}_{W_4}$ is chiral invariant, i.e. is invariant under *global* $U(1)$ transformations ($\beta' = \text{const}$):

$$\psi_L \rightarrow e^{-i\beta'} \psi_L; \quad \psi_R \rightarrow e^{i\beta'} \psi_R; \quad \phi \rightarrow e^{-2i\beta'} \phi, \quad (2.31)$$

and analogously for $\bar{\psi}_L, \bar{\psi}_R$ and ϕ^\dagger with the complex conjugate phase factors.

The field strengths (curvatures) entering the expression (2.29) in addition to $f_{\mu\nu}$ and the Weyl curvature scalar R defined by [see (I, A31)]

$$R = \bar{R} - 3\bar{\nabla}^\rho\kappa_\rho + \frac{3}{2}\kappa^\rho\kappa_\rho, \quad (2.32)$$

where \bar{R} is the Riemannian part, are the $U(1)_Y$ gauge curvature

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (2.33)$$

and the $SU(2)_W$ i.e. Yang-Mills gauge curvature

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \tilde{g} f_{bc}^a A_\mu^b A_\nu^c, \quad (2.34)$$

with $f_{bc}^a = \varepsilon_{abc}$ denoting the structure constants of $SU(2)_W$ where ε_{abc} is the Levi-Civita symbol. The overall constant K in (2.29) with dimension $[\text{Energy} \cdot L^{-1}]$ is a factor converting the length dimension in the curly brackets (which is $[L^{-2}]$) into $[\text{Energy} \cdot L^{-3}]$ in order to give $\tilde{\mathcal{L}}_{W_4}$ — finally, after symmetry breaking — the correct dimension of an energy density. This factor K drops out of the field equations in the Weyl-symmetric case discussed in this section and appears in (2.29) only for convenience. We finally remark that the length dimension of the scalar field ϕ is assumed to be $[L^0]$ and relative to this choice the fermion fields have length dimension $[L^{-\frac{1}{2}}]$. With this convention the Yukawa coupling constant $\tilde{\gamma}$ has length dimension $[L^{-1}]$.

So far we have not included a coupling $\sim \sqrt{-g} F_{\mu\nu} f^{\mu\nu}$ in the Lagrangean (I, 3.8) or a coupling $\sim \sqrt{-g} B_{\mu\nu} f^{\mu\nu}$ in (2.29) which would also be of Weyl weight zero and hence would be allowed to occur. We intend to come back to an investigation of this point in a separate context and restrict the discussion here to the direct product structure of the abelian gauge groups involved, treating them thus as completely independent from each another.

The variation of the fields in the Lagrangean (2.29) now yields the following set of G -covariant field equations:

$$\delta\phi^\dagger : \quad g^{\mu\nu} \tilde{D}_\mu \tilde{D}_\nu \phi + \frac{1}{6} R \phi + 4\beta(\phi^\dagger \phi) \phi - 2\tilde{\gamma} \bar{\psi}_R \psi_L = 0, \quad (2.35)$$

$$\delta\psi_L^\dagger : \quad -i\gamma^\mu \tilde{D}_\mu \psi_L - \tilde{\gamma} \phi \psi_R = 0, \quad (2.36)$$

$$\delta\psi_R^\dagger : \quad -i\gamma^\mu \tilde{D}_\mu \psi_R - \tilde{\gamma}(\phi^\dagger \psi_L) = 0, \quad (2.37)$$

$$\delta\kappa_\rho : \quad \tilde{\delta} D_\mu f^{\mu\rho} = -6\tilde{\alpha} D^\rho R, \quad (2.38)$$

$$\delta B_\rho : \quad \tilde{\delta} D_\mu B^{\mu\rho} = \tilde{g}' [j^{(\phi)\rho} + j^{(\psi_L)\rho} + j^{(\psi_R)\rho}], \quad (2.39)$$

$$\delta A_\rho^a : \quad \tilde{\delta} \tilde{D}_\mu F_a^{\mu\rho} \equiv \tilde{\delta} [D_\mu F_a^{\mu\rho} - \tilde{g} f_{ac}^b A_\mu^c F_b^{\mu\rho}] = \tilde{g} [j_a^{(\phi)\rho} + j_a^{(\psi_L)\rho}], \quad (2.40)$$

$$\begin{aligned} \delta g^{\mu\nu} : \quad & \frac{1}{6} \Phi^2 \left[R_{(\mu\nu)} - \frac{1}{2} g_{\mu\nu} R \right] - 4\tilde{\alpha} R \left[R_{(\mu\nu)} - \frac{1}{4} g_{\mu\nu} R \right] - 4\tilde{\alpha} \{ D_{(\mu} D_{\nu)} R - g_{\mu\nu} D^\rho D_\rho R \} = \\ & = \Theta_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(\psi_L)} + T_{\mu\nu}^{(\psi_R)} + T_{\mu\nu}^{(f)} + T_{\mu\nu}^{(B)} + T_{\mu\nu}^{(F_a)} - g_{\mu\nu} \tilde{\gamma} [(\bar{\psi}_L \phi) \psi_R + \bar{\psi}_R (\phi^\dagger \psi_L)]. \end{aligned} \quad (2.41)$$

Here we have used the following hermitean and G -gauge covariant expressions for the weak hypercharge and isospin source currents:

$$j_\rho^{(\phi)} = \frac{i}{4} \left(\phi^\dagger \overleftrightarrow{\tilde{D}}_\rho \phi - \phi^\dagger \overleftarrow{\tilde{D}}_\rho \phi \right) \quad (2.42)$$

for the $U(1)_Y$ ϕ -current (with $Y = \frac{1}{2}$ for the ϕ -field),

$$j_\rho^{(\psi_L)} = -\frac{1}{2} (\bar{\psi}_L \gamma_\rho \psi_L) ; \quad j_\rho^{(\psi_R)} = -(\bar{\psi}_R \gamma_\rho \psi_R), \quad (2.43)$$

for the left- and right-handed $U(1)_Y$ fermion currents (with $Y = -\frac{1}{2}$ and $Y = -1$, respectively), and for the $SU(2)_W$ currents

$$j_{a\rho}^{(\phi)} = \frac{i}{2} \left(\phi^\dagger \frac{1}{2} \tau_a \overrightarrow{\widetilde{D}}_\rho \phi - \phi^\dagger \overleftarrow{\widetilde{D}}_\rho \frac{1}{2} \tau_a \phi \right), \quad (2.44)$$

$$j_{a\rho}^{(\psi_L)} = \bar{\psi}_L \gamma_\rho \frac{1}{2} \tau_a \psi_L; \quad j_{a\rho}^{(\psi_R)} \equiv 0. \quad (2.45)$$

The G -gauge covariant expressions for the symmetric energy-momentum tensors appearing in (2.41) are:

$$\begin{aligned} \Theta_{\mu\nu}^{(\phi)} = \frac{1}{2} \left[(\widetilde{D}_\mu \phi)^\dagger (\widetilde{D}_\nu \phi) + (\widetilde{D}_\nu \phi)^\dagger (\widetilde{D}_\mu \phi) \right] + \frac{1}{6} \left\{ D_{(\mu} D_{\nu)} \Phi^2 - g_{\mu\nu} D^\rho D_\rho \Phi^2 \right\} \\ - g_{\mu\nu} \left[\frac{1}{2} g^{\rho\lambda} (\widetilde{D}_\rho \phi)^\dagger (\widetilde{D}_\lambda \phi) - \beta (\phi^\dagger \phi)^2 \right], \end{aligned} \quad (2.46)$$

$$T_{\mu\nu}^{(\psi_L)} = \frac{i}{2} \left\{ \bar{\psi}_L \gamma_{(\mu} \overrightarrow{\widetilde{D}}_{\nu)} \psi_L - \bar{\psi}_L \overleftarrow{\widetilde{D}}_{(\mu} \gamma_{\nu)} \psi_L \right\} - g_{\mu\nu} \frac{i}{2} \left\{ \bar{\psi}_L \gamma^\rho \overrightarrow{\widetilde{D}}_\rho \psi_L - \bar{\psi}_L \overleftarrow{\widetilde{D}}_\rho \gamma^\rho \psi_L \right\}, \quad (2.47)$$

and analogously for $T_{\mu\nu}^{(\psi_R)}$, and

$$T_{\mu\nu}^{(f)} = -\tilde{\delta} \left[f_{\mu\sigma} f_\nu^\sigma - \frac{1}{4} g_{\mu\nu} f^{\rho\lambda} f_{\rho\lambda} \right], \quad (2.48)$$

$$T_{\mu\nu}^{(B)} = -\tilde{\delta} \left[B_{\mu\sigma} B_\nu^\sigma - \frac{1}{4} g_{\mu\nu} B^{\rho\lambda} B_{\rho\lambda} \right], \quad (2.49)$$

$$T_{\mu\nu}^{(F_a)} = -\tilde{\delta} \left[F_{\mu\sigma}^a F_{\nu}^\sigma - \frac{1}{4} g_{\mu\nu} F_a^{\rho\lambda} F_{\rho\lambda}^a \right]. \quad (2.50)$$

While the last three energy-momentum tensors have vanishing trace the traces of $\Theta_{\mu\nu}^{(\phi)}$, $T_{\mu\nu}^{(\psi_L)}$ and $T_{\mu\nu}^{(\psi_R)}$ for solutions of the field equations (2.35) – (2.37) and their adjoints read:

$$\Theta_\mu^{(\phi)\mu} = -\frac{1}{6} R \phi^\dagger \phi + \tilde{\gamma} \left[(\bar{\psi}_L \phi) \psi_R + \bar{\psi}_R (\phi^\dagger \psi_L) \right], \quad (2.51)$$

$$T_\mu^{(\psi_L)\mu} = T_\mu^{(\psi_R)\mu} = \frac{3}{2} \tilde{\gamma} \left[(\bar{\psi}_L \phi) \psi_R + \bar{\psi}_R (\phi^\dagger \psi_L) \right]. \quad (2.52)$$

Using these relations in computing the trace of Eq. (2.41), together with $D^\rho D_\rho R = 0$ following from (2.38), one finds that the resulting equation is identically satisfied.

It is also interesting to compute the Weyl-covariant divergences of the energy-momentum tensors (2.46) – (2.50) for solutions of the field equations, i.e. using Eqs. (2.35) – (2.40) as well as the Bianchi identities for the $F_{\mu\nu}^a$ reading, with $\{\rho\mu\nu\}$ denoting the cyclic sum of the indices $\rho\mu\nu$,

$$\tilde{\widetilde{D}}_{\{\rho} F_{\mu\nu\}}^a = 0. \quad (2.53)$$

The result is

$$D^\mu \Theta_{\mu\nu}^{(\phi)} = \frac{1}{6} \left[R_{(\nu\rho)} - \frac{1}{2} g_{\nu\rho} R \right] D^\rho \Phi^2 - \frac{1}{12} \Phi^2 D^\rho f_{\rho\nu} + \tilde{\gamma} \left[(\bar{\psi}_L \psi_R \overrightarrow{\widetilde{D}}_\nu \phi) + (\phi^\dagger \overleftarrow{\widetilde{D}}_\nu \bar{\psi}_R \psi_L) \right], \quad (2.54)$$

$$D^\mu T_{\mu\nu}^{(\psi_L)} = \tilde{\gamma} Y_\nu(\phi, \psi_L, \psi_R) + \tilde{g}' B_{\nu\rho} j^{(\psi_L)\rho} + \tilde{g} F_{\nu\rho}^a j_a^{(\psi_L)\rho}, \quad (2.55)$$

$$D^\mu T_{\mu\nu}^{(\psi_R)} = \tilde{\gamma} Y_\nu(\phi, \psi_L, \psi_R) + \tilde{g}' B_{\nu\rho} j^{(\psi_R)\rho}, \quad (2.56)$$

with the contribution $Y_\nu(\phi, \psi_L, \psi_R)$ of the Yukawa coupling to Eqs. (2.55) and (2.56):

$$Y_\nu(\phi, \psi_L, \psi_R) = \frac{1}{4} \left[\left\{ \bar{\psi}_L(\phi^\dagger \overleftrightarrow{D}_\nu \psi_L) + (\bar{\psi}_L \overleftrightarrow{D}_\nu \phi) \psi_R \right\} - \left\{ (\bar{\psi}_L \phi) \overleftrightarrow{D}_\nu \psi_R + \bar{\psi}_R \overleftrightarrow{D}_\nu (\phi^\dagger \psi_L) \right\} \right. \\ \left. + \left\{ (\psi_L \psi_R \overleftrightarrow{D}_\nu \phi) + (\phi^\dagger \overleftrightarrow{D}_\nu \bar{\psi}_R \psi_L) \right\} - \left\{ (\bar{\psi}_L \gamma_\nu \gamma_\rho \psi_R \overleftrightarrow{D}^\rho \phi) + (\phi^\dagger \overleftrightarrow{D}^\rho \bar{\psi}_R \gamma_\rho \gamma_\nu \psi_L) \right\} \right], \quad (2.57)$$

and

$$D^\mu T_{\mu\nu}^{(f)} = 6\tilde{\alpha} f_{\nu\rho} D^\rho R, \quad (2.58)$$

$$D^\mu T_{\mu\nu}^{(B)} = -\tilde{g}' B_{\nu\rho} \left[j^{(\phi)\rho} + j^{(\psi_L)\rho} + j^{(\psi_R)\rho} \right], \quad (2.59)$$

$$D^\mu T_{\mu\nu}^{(F_a)} = -\tilde{g} F_{\nu\rho}^a \left[j_a^{(\phi)\rho} + j_a^{(\psi_L)\rho} \right]. \quad (2.60)$$

The last two equations as well as the field equations (2.39) and (2.40) show that it is useful to introduce the following total $U(1)_Y$ and total $SU(2)_W$ current densities of Weyl weight zero composed of bosonic and fermionic contributions:

$$U(1)_Y \quad : \quad J_\rho = -\frac{\partial \tilde{\mathcal{L}}_{W_4}}{\partial B^\rho} = K\sqrt{-g} \tilde{g}' \left[j_\rho^{(\phi)} + j_\rho^{(\psi_L)} + j_\rho^{(\psi_R)} \right], \quad (2.61)$$

$$SU(2)_W \quad : \quad J_{a\rho} = -\frac{\partial \tilde{\mathcal{L}}_{W_4}}{\partial A^{a\rho}} = K\sqrt{-g} \tilde{g} \left[j_{a\rho}^{(\phi)} + j_{a\rho}^{(\psi_L)} \right]. \quad (2.62)$$

These definitions allow the separation of the Lagrangean density $\tilde{\mathcal{L}}_{W_4}$ into a Weyl-invariant “free” part $\tilde{\mathcal{L}}_{W_4}^{(0)}$ (obtained from Eq. (2.29) for $B_\rho \equiv 0$ and $A_\rho^a \equiv 0$) and a Weyl-invariant interaction part $\tilde{\mathcal{L}}_{int}$ expressed in terms of the currents (2.61) and (2.62) and the corresponding gauge fields, i.e.

$$\tilde{\mathcal{L}}_{int} = -J^\rho B_\rho - J_a^\rho A_\rho^a, \quad (2.63)$$

yielding thus the decomposition

$$\tilde{\mathcal{L}}_{W_4} = \tilde{\mathcal{L}}_{W_4}^{(0)} + \tilde{\mathcal{L}}_{int} + \tilde{\mathcal{L}}^{(B, F_a)}, \quad (2.64)$$

where $\tilde{\mathcal{L}}^{(B, F_a)}$ is given by the last two terms in (2.29) proportional to $\tilde{\delta}$ representing the Lagrangean density of the free B_ρ and A_ρ^a fields.

The energy-momentum balance for the interacting fields is represented by the Weyl-covariant divergence of Eq. (2.41). Using Eqs. (2.54)–(2.60) together with the contracted Bianchi identities (I, A40) for a W_4 , it is seen that this is identically satisfied for solutions of the field equations for any value of $\tilde{\gamma}$. We shall find later in Sect. III that the analogous argument for the broken Weyl theory yields a constraint which is required to be satisfied for the divergence relation following from the $g_{\mu\nu}$ -equations — representing the over-all energy-momentum conservation in the broken case — to vanish again.

Let us now turn to the conservation relations for the currents defined in Eqs. (2.42)–(2.45) as well as for the total electromagnetic current. Introducing the charge operator

$$\hat{q} = \frac{1}{2}(1 + \tau_3) \quad (2.65)$$

for the ϕ -field the total electromagnetic current is given by

$$\begin{aligned} j_\rho^{(e.m.)} &= \frac{1}{K\sqrt{-g}} \left(\frac{1}{\tilde{g}'} J_\rho + \frac{1}{\tilde{g}} J_{3\rho} \right) \equiv j_\rho^{(\phi)} + j_\rho^{(\psi_L)} + j_\rho^{(\psi_R)} + j_{3\rho}^{(\phi)} + j_{3\rho}^{(\psi_L)} \\ &= \frac{i}{2} \left(\phi^\dagger \hat{q} \overleftrightarrow{D}_\rho \phi - \phi^\dagger \overleftrightarrow{D}_\rho \hat{q} \phi \right) - \bar{\psi}_e \gamma_\rho \psi_e, \end{aligned} \quad (2.66)$$

Here \hat{q} projects out the φ_+ -component of ϕ with charge $q = +1e$ in the ϕ -part of the electromagnetic current while the last term in (2.66) yields, with $-(\bar{e}_L \gamma_\rho e_L) - (\bar{e}_R \gamma_\rho e_R) = -\bar{\psi}_e \gamma_\rho \psi_e$, the electromagnetic current contribution of the charged lepton (electron) with $q = -1e$.

It is easy to show from the field equations (2.36) and (2.37) and their adjoints that the Weyl-covariant divergence of the currents (2.43) yield

$$D^\rho j_\rho^{(\psi_R)} = -2D^\rho j_\rho^{(\psi_L)} = i\tilde{\gamma} \left[(\bar{\psi}_L \phi) \psi_R - \bar{\psi}_R (\phi^\dagger \psi_L) \right]. \quad (2.67)$$

This implies for the fermion part of the electromagnetic current the relation

$$\tilde{D}^\rho \left[j_\rho^{(\psi_L)} + j_\rho^{(\psi_R)} + j_{3\rho}^{(\psi_L)} \right] = i\tilde{\gamma} \left[(\bar{\psi}_L \hat{q} \phi) \psi_R - \bar{\psi}_R (\phi^\dagger \hat{q} \psi_L) \right], \quad (2.68)$$

where we have, moreover, used (for $a = 3$) the divergence relation for the fermionic isospin current

$$\tilde{D}^\rho j_{a\rho}^{(\psi_L)} \equiv D^\rho j_{a\rho}^{(\psi_L)} - \tilde{g} \varepsilon_{abc} A_\rho^b j_c^{(\psi_L)\rho} = i\tilde{\gamma} \left[(\bar{\psi}_L \frac{1}{2} \tau_a \phi) \psi_R - \bar{\psi}_R (\phi^\dagger \frac{1}{2} \tau_a \psi_L) \right] \quad (2.69)$$

following from Eqs. (2.45) and (2.36). On the other hand, one concludes from Eqs. (2.39) and (2.40) that the hypercharge and isospin currents appearing on the r.-h. sides of these equations are covariantly conserved.

Similarly, one concludes from the field equation (2.35) and its adjoint that the ϕ -part of the isospin current obeys

$$\tilde{D}^\rho j_{a\rho}^{(\phi)} = -i\tilde{\gamma} \left[(\bar{\psi}_L \frac{1}{2} \tau_a \phi) \psi_R - \bar{\psi}_R (\phi^\dagger \frac{1}{2} \tau_a \psi_L) \right], \quad (2.70)$$

yielding for the sum $j_\rho^{(\phi)} + j_{3\rho}^{(\phi)}$ the relation

$$\tilde{D}^\rho \left[j_\rho^{(\phi)} + j_{3\rho}^{(\phi)} \right] = -i\tilde{\gamma} \left[(\bar{\psi}_L \hat{q} \phi) \psi_R - \bar{\psi}_R (\phi^\dagger \hat{q} \psi_L) \right]. \quad (2.71)$$

Eqs. (2.68) and (2.71) together, finally, lead for the total electromagnetic current (2.66) to the conservation relation

$$D^\rho j_\rho^{(e.m.)} = 0, \quad (2.72)$$

where, for a fixed 3-direction, we have replaced the $SU(2)_W$ and Weyl-covariant derivative \tilde{D}_ρ by D_ρ .

As usual we now introduce the Weinberg angle θ_W by the rotation relating $A_{3\rho}$ and B_ρ to the $q = 0$ component of the the $SU(2)_W$ gauge fields Z_ρ and the electromagnetic fields A_ρ :

$$\begin{aligned} A_{3\rho} &= \cos \theta_W Z_\rho + \sin \theta_W \frac{e}{\hbar c} A_\rho \\ B_\rho &= -\sin \theta_W Z_\rho + \cos \theta_W \frac{e}{\hbar c} A_\rho. \end{aligned} \quad (2.73)$$

The factor $e/\hbar c$ in front of the photon field is introduced here for dimensional reasons with the fields $A_{3\rho}, B_\rho, Z_\rho$, and $\frac{e}{\hbar c} A_\rho$ all having length dimension $[L^{-1}]$. Below we shall frequently abbreviate the electromagnetic potential $\frac{e}{\hbar c} A_\mu$ with dimension $[L^{-1}]$ as \bar{A}_μ . The dimensionless gauge coupling constants \tilde{g} and \tilde{g}' were introduced in Eq. (2.28). In addition we shall introduce below the dimensionless coupling constant \tilde{g}_0 for the neutral fields Z_ρ and a dimensionless strength for the electromagnetic coupling given by $\tilde{e} = \sqrt{4\pi\alpha_F}$ with $\alpha_F = e^2/\hbar c = 1/137.04$ denoting the fine-structure constant. As usual the elementary electromagnetic charge — a quantity with a dimension — is denoted by e with $-|e| = -4.8032 \cdot 10^{-10} esu$ being the electron charge. [$1 esu = 1 dyn^{\frac{1}{2}} cm$; we are using *cgs*-units as conventional reference units. The intrinsic length unit obtained after Weyl-symmetry breaking will be introduced in Sect. III below.]

Using now a spherical basis for the isovector contributions in Eq. (2.63) introducing the following charge changing current components with $\Delta q = \pm 1$ [compare (2.62)]

$$J_\rho^{(1)} = \frac{1}{\sqrt{2}} (J_{1\rho} - iJ_{2\rho}); \quad J_\rho^{(1)\dagger} = \frac{1}{\sqrt{2}} (J_{1\rho} + iJ_{2\rho}), \quad (2.74)$$

and the corresponding gauge fields

$$W_\rho = \frac{1}{\sqrt{2}} (A_{1\rho} - iA_{2\rho}); \quad W_\rho^\dagger = \frac{1}{\sqrt{2}} (A_{1\rho} + iA_{2\rho}), \quad (2.75)$$

and expressing, furthermore, the fields $A_{3\rho}$ and B_ρ in terms of the fields Z_ρ and A_ρ according to (2.73) the interaction Lagrangean (2.63) of Weyl weight zero may be written as

$$\tilde{\mathcal{L}}_{int} = -K\sqrt{-g} \left(\tilde{e} j_\rho^{(e.m.)} \bar{A}^\rho + \tilde{g}_0 j_\rho^{(0)} Z^\rho \right) - \left(J_\rho^{(1)\dagger} W^\rho + J_\rho^{(1)} W^{\rho\dagger} \right). \quad (2.76)$$

Here $j_\rho^{(e.m.)}$ is the electromagnetic current defined in (2.66). In order to obtain the electromagnetic interaction in the conventional form given in (2.76) one has to demand that the coupling constants \tilde{g} and \tilde{g}' and the Weinberg angle θ_W are related by

$$\tilde{g} \sin \theta_W = \tilde{g}' \cos \theta_W = \frac{\tilde{g}\tilde{g}'}{\sqrt{\tilde{g}^2 + \tilde{g}'^2}} = \tilde{e}, \quad (2.77)$$

where \tilde{e} is the dimensionless electromagnetic coupling strength introduced above. The second term in the first bracket of (2.76) describes the coupling of the neutral weak current $j_\rho^{(0)}$ to the neutral gauge fields Z_ρ with a coupling strength given by

$$\tilde{g}_0 = \frac{\tilde{g}\tilde{g}'}{\tilde{e}} = \frac{\tilde{g}}{\cos \theta_W} = \sqrt{\tilde{g}^2 + \tilde{g}'^2}, \quad (2.78)$$

where $j_\rho^{(0)}$ is defined by

$$j_\rho^{(0)} = j_{3\rho}^{(\phi)} + j_{3\rho}^{(\psi_L)} - \sin^2 \theta_W j_\rho^{(e.m.)}. \quad (2.79)$$

All these definitions are the same as those appearing in the usual formulation of the standard electroweak model.

Along with $J_\rho^{(1)}$ and $J_\rho^{(1)\dagger}$ one may now also introduce the current densities $J_\rho^{(e.m.)}$ and $J_\rho^{(0)}$ of Weyl weight zero by

$$J_\rho^{(e.m.)} = K\sqrt{-g} \tilde{e} j_\rho^{(e.m.)} ; \quad J_\rho^{(0)} = K\sqrt{-g} \tilde{g}_0 j_\rho^{(0)}. \quad (2.80)$$

The last two terms in Eq. (2.76) represent the coupling of the charged $SU(2)_W$ gauge fields to the charge changing weak currents describing the weak decays of particles in phenomenological applications. For low momentum transfer processes, $p^2 \approx 0$, we have by identification with the effective current-current theory of weak interactions with the Fermi constant G_F the relation

$$\left(\frac{1}{2} \frac{\tilde{g}}{\sqrt{2}} \right)^2 \frac{1}{m_W^2} = \frac{G_F}{\sqrt{2}}, \quad (2.81)$$

where m_W is the mass of the W -field defined in Eq. (3.50) below. Compare in this context also Eq. (2.62) as well as (3.32) below.

C. Remarks Concerning Symmetry Breaking

We, finally, compute the square of the Dirac operators

$$\tilde{\not{D}} = -i\gamma^\mu \tilde{D}_\mu \quad \text{and} \quad \not{\tilde{D}} = -i\gamma^\mu \tilde{D}_\mu \quad (2.82)$$

in application to the fermion fields ψ_L and ψ_R , respectively. From the field equations (2.36) and (2.37) one easily derives that

$$\tilde{\not{D}} \not{\tilde{D}} \psi_L = -i\tilde{\gamma}\gamma^\mu \psi_R \tilde{D}_\mu \phi + \tilde{\gamma}^2 \phi(\phi^\dagger \psi_L), \quad (2.83)$$

and

$$\not{\tilde{D}} \tilde{\not{D}} \psi_R = -i\tilde{\gamma}(\phi^\dagger \overleftarrow{\tilde{D}}_\mu \gamma^\mu \psi_L) + \tilde{\gamma}^2 (\phi^\dagger \phi) \psi_R. \quad (2.84)$$

As regards the $SU(2)_W$ degrees of freedom Eq. (2.83) is a matrix equation for a two-component isospinor while (2.84) is an equation for an isoscalar with the round brackets denoting $SU(2)_W$ invariants. The left- and right-handed fields are coupled in these equations through the first terms on their r.-h. sides. We observe that both equations *decouple* and become eigenvalue equations for ψ_L and ψ_R , respectively, which are moreover *diagonal in spin space* provided the ϕ -field is covariant constant, i.e. obeys

$$\tilde{D}_\mu \phi = 0. \quad (2.85)$$

A property of this type would annihilate the first term in the field equations (2.36) for ϕ and would yield an algebraic constraint involving the curvature scalar R , the ϕ -field, and

the ψ -fields. It is not immediately clear whether this is consistent with the other field equations. Therefore we shall not demand Eq. (2.85) to be satisfied. However, an equation of a similar nature will be used in Sect. III below when we investigate the breaking of the Weyl symmetry to obtain, finally, a gauge theory formulated in a Riemannian space-time in the limit.

After these remarks concerning the standard electroweak theory and its formulation in a Weyl geometric framework we turn, as mentioned, to the breaking of the G -gauge symmetry. Let us, however, first consider, at the end of this section, the so-called breaking of the electroweak gauge symmetry $\tilde{G} = SU(2)_W \times U(1)_Y$ to the electromagnetic gauge symmetry $U(1)_{e.m.}$ which is described in the standard model as the result of a so-called “spontaneous symmetry breaking due to a nonvanishing vacuum expectation value of the scalar field”.

In order to view the situation more clearly we investigate in Appendix A the coset representation for the field ϕ in terms of transformations $U(\bar{g}_\phi)$ or $U(\phi)$ parametrizing \tilde{G}/H [see (A5) and (A11)] by generating the function ϕ from the real function $\hat{\phi} = \begin{pmatrix} 0 \\ \varphi_0 \end{pmatrix}$ with $\hat{\phi}$ being invariant under the subgroup H of \tilde{G} , where H is identified with the *electromagnetic* gauge group $U(1)_{e.m.} = U(1)_+$ generated by \hat{q} and obeying $\hat{q}\hat{\phi} = 0$ [compare (A3) and (A7)]. The transition from ϕ to $\hat{\phi}$ with the help of the transformation $U^{-1}(\phi)$ is to be regarded as a *choice of coordinates* for the representation of the scalar field in the theory and has, in the first place, nothing to do with a “vacuum expectation value” of this field. To adopt the origin $\hat{\phi}$ in \tilde{G}/H as a parametrization for the scalar field is done for physical reasons establishing thereby the electromagnetic gauge group $H = U(1)_{e.m.}$ as stability group of the point $\hat{\phi}$ in the formalism and relate it to physical observations and experiments. This choice is, actually, not a breaking of the original \tilde{G} -gauge theory but a different realization of it. It is thus better to say that after transforming ϕ to $\hat{\phi}$ with the help of $U^{-1}(\phi)$ one has adopted an *electromagnetic gauge* in the electroweak theory with the residual gauge transformations being given by $U(h(\alpha)) \in U(1)_{e.m.}$ [see (A7), (A15) and the relation of these transformations to the so-called “Wigner rotations”]. A true symmetry reduction from G to a subgroup of G is governed by a relation of the type (2.85) to which we turn in the next section [6].

Conditions of the type of Eq. (2.85) where ϕ is a section on a bundle with a homogeneous space as fiber are well-known from differential geometry. They guarantee that in a reduction of the structural group of a certain principal bundle $P(B, G)$ over the base B and with structural group G to a bundle $P'(B, G')$ over the same base and with a subgroup G' of G as structural group also the corresponding *connection* reduces from a \mathfrak{g} -valued to a \mathfrak{g}' -valued form, where \mathfrak{g} and \mathfrak{g}' denote the respective Lie algebras of the groups G and $G' \subset G$ (compare [7] for details). In concluding this section let us, therefore, state the following theorem well-known from the literature on differential geometry: *As the condition for a true symmetry reduction in the physical sense from a theory with gauge group G to a theory with gauge group $G' \subset G$, implying also the reduction of the connection on $P(B, G)$ to the connection on $P'(B, G')$, it is required that there exists a section ϕ_E on the bundle $E(B, G/G', G)$, associated to $P(B, G)$, with fiber $F = G/G'$ and structural group G which is covariant constant, i.e. obeys $D\phi_E = 0$, where D is the covariant derivative on E .*

III. WEYL-SYMMETRY BREAKING

As a term in the Lagrangean which breaks the Weyl-symmetry with the aim of introducing a scale of lengths with the help of the ϕ -field and establish an electroweak theory of leptons in the presence of gravitation formulated in a V_4 , we add the following expression of Weyl weight +1 to the Weyl-invariant Lagrangean density $\tilde{\mathcal{L}}_{W_4}$ given in (2.29):

$$\mathcal{L}_B = -\frac{a}{2}K\sqrt{-g}\left\{\frac{1}{6}R + \left[\frac{mc}{\hbar}\right]^2\phi^\dagger\phi\right\}. \quad (3.1)$$

Here a is a dimensionless constant, R is the curvature scalar of the ambient Weyl space W_4 [see (2.32)] which is related here to the mass — or rather Compton wave length — of the universal scalar field ϕ by tying R to the squared modulus $\Phi^2 = \phi^\dagger\phi$ of this field. The expression (3.1) is independent of the standard model gauge fields associated with the group \tilde{G} and thus leaves the \tilde{G} -gauge invariance unaffected. However, the explicit breaking of the $D(1)$ symmetry caused by \mathcal{L}_B will lead, as we shall see, to nonzero masses not only for the ϕ -field but also for the fermion and the gauge boson fields in a manner similar to the situation realized in the standard electroweak theory.

We have mentioned at the end of Sect. II that the standard model is not characterized by a true symmetry reduction from a gauge group \tilde{G} to a subgroup of \tilde{G} . What is conventionally called a spontaneous symmetry breaking is a choice of an appropriate coordinatization taking due recognition of the electromagnetic phase group $U(1)_{e.m.}$ as a subgroup generated by \hat{q} in the formalism. On the contrary, adding (3.1) to the Lagrangean $\tilde{\mathcal{L}}_{W_4}$ will lead to a true breaking of the G -gauge theory [see (2.16)] to a theory with the subgroup $G' = SO(3,1) \otimes \tilde{G}$ of G as gauge group. It is the square of the modulus, Φ^2 , of the ϕ -field which is the section on $E(W_4, G/G', G)$, required to be covariant constant in the sense of the theorem quoted at the end of Sect. II, which governs, as we shall see, the symmetry breaking by (3.1) yielding, ultimately, a V_4 from a W_4 and the generation of nonvanishing masses. Hence the symmetry breaking relation will be

$$D_\mu\Phi^2 \equiv \partial_\mu\Phi^2 + \kappa_\mu\Phi^2 = 0, \quad (3.2)$$

with $\Phi^2 \in G/G' \equiv D(1)$.

However, before we come to this point, let us first derive the field equations following from a variational principle formulated with the Lagrangean \mathcal{L} given by

$$\mathcal{L} = \tilde{\mathcal{L}}_{W_4} + \mathcal{L}_B. \quad (3.3)$$

One finds using the same notation as above:

$$\delta\phi^\dagger : \quad g^{\mu\nu} \tilde{D}_\mu \tilde{D}_\nu \phi + \frac{1}{6}R\phi + 4\beta(\phi^\dagger\phi)\phi - 2\tilde{\gamma}\bar{\psi}_R\psi_L + a\left[\frac{mc}{\hbar}\right]^2\phi = 0, \quad (3.4)$$

$$\delta\psi_L^\dagger : \quad -i\gamma^\mu \tilde{D}_\mu \psi_L - \tilde{\gamma}\phi\psi_R = 0, \quad (3.5)$$

$$\delta\psi_R^\dagger : \quad -i\gamma^\mu \tilde{D}_\mu \psi_R - \tilde{\gamma}(\phi^\dagger\psi_L) = 0, \quad (3.6)$$

$$\delta\kappa_\rho : \quad \tilde{\delta}D_\mu f^{\mu\rho} = -6\tilde{\alpha}D^\rho R + \frac{a}{4}\kappa^\rho, \quad (3.7)$$

$$\delta B_\rho : \quad \tilde{\tilde{\delta}}D_\mu B^{\mu\rho} = \tilde{g}' \left[j^{(\phi)\rho} + j^{(\psi_L)\rho} + j^{(\psi_R)\rho} \right], \quad (3.8)$$

$$\delta A_\rho^a : \quad \tilde{\tilde{\delta}}\tilde{D}_\mu F_a^{\mu\rho} = \tilde{g} \left[j_a^{(\phi)\rho} + j_a^{(\psi_L)\rho} \right], \quad (3.9)$$

$$\begin{aligned} \delta g_{\mu\nu} : \quad & \frac{1}{6}(\Phi^2 + a) \left[R_{(\mu\nu)} - \frac{1}{2}g_{\mu\nu}R \right] - 4\tilde{\alpha}R \left[R_{(\mu\nu)} - \frac{1}{4}g_{\mu\nu}R \right] - 4\tilde{\alpha} \left\{ D_{(\mu}D_{\nu)}R - g_{\mu\nu}D^\rho D_\rho R \right\} = \\ & = \Theta_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(\psi_L)} + T_{\mu\nu}^{(\psi_R)} + T_{\mu\nu}^{(f)} + T_{\mu\nu}^{(B)} + T_{\mu\nu}^{(F_a)} - g_{\mu\nu}\tilde{\gamma} \left[(\bar{\psi}_L\phi)\psi_R + (\bar{\psi}_R(\phi^\dagger\psi_L)) \right] + \\ & \quad + g_{\mu\nu}\frac{a}{2} \left[\frac{mc}{\hbar} \right]^2 \Phi^2, \end{aligned} \quad (3.10)$$

$$\delta a : \quad \frac{1}{6}R + \left[\frac{mc}{\hbar} \right]^2 \Phi^2 = 0, \quad (3.11)$$

where Eqs. (3.5), (3.6), (3.8) and (3.9) are unchanged; compare Eqs. (2.36), (2.37), (2.39) and (2.40). The energy-momentum tensors appearing in (3.10) are the same as those defined in Eqs. (2.46) – (2.50) of Sect. II. Only the energy-momentum tensor for the scalar field is to be changed now, for $a \neq 0$, to the expression given by the sum of the first and the last term on the r.-h. side of (3.10), i.e. by

$$\Theta_{\mu\nu}^{(\phi)'} = \Theta_{\mu\nu}^{(\phi)} + g_{\mu\nu} \frac{a}{2} \left[\frac{mc}{\hbar} \right]^2 \Phi^2. \quad (3.12)$$

We first turn to the trace condition following from (3.10). The trace of $\Theta_{\mu\nu}^{(\phi)'}$ for solutions of the field equation (3.4) and its adjoint is now given by

$$\Theta_\mu^{(\phi)'}{}^\mu = -\frac{1}{6}R\phi^\dagger\phi + \tilde{\gamma} \left[(\bar{\psi}_L\phi)\psi_R + \bar{\psi}_R(\phi^\dagger\psi_L) \right] + a \left[\frac{mc}{\hbar} \right]^2 \phi^\dagger\phi, \quad (3.13)$$

while the other traces of the energy-momentum tensors are the same as in Sect. II. With these and Eq. (3.13) one concludes from the trace of (3.10) that

$$\tilde{\alpha}D^\rho D_\rho R = 0. \quad (3.14)$$

Taking the Weyl-covariant divergence of Eq. (3.7) one finds with (3.14) that the Weyl vector fields must satisfy the Lorentz-like condition:

$$D_\rho\kappa^\rho \equiv \bar{\nabla}_\rho\kappa^\rho - \kappa_\rho\kappa^\rho = 0 \quad \text{for} \quad a \neq 0, \quad (3.15)$$

where $\bar{\nabla}_\rho$ denotes the metric covariant derivative [compare Appendix A of I]. Eq. (3.15) in turn implies that the W_4 curvature scalar (2.32) is now given by

$$R = \bar{R} - \frac{3}{2}\kappa_\rho\kappa^\rho. \quad (3.16)$$

We, finally, compute the divergence conditions for the solutions of the field equations (3.4) – (3.11) which follow from (3.10) by taking the Weyl-covariant divergence of this equation and using the contracted Bianchi identities (I, A40) for the W_4 as well as the equations

$$D^\mu T_{\mu\nu}^{(f)} = f_\nu{}^\rho \left[6\tilde{\alpha} D_\rho R - \frac{a}{4} \kappa_\rho \right]. \quad (3.17)$$

Before the symmetry breaking by \mathcal{L}_B these energy-momentum balance relations for the set of interacting fields were identically satisfied in Sect. II. In the broken case we now obtain that the following relations must hold for the divergence relations, deduced from (3.10), to be fulfilled again [compare (I, 4.16)] :

$$D^\mu f_{\mu\nu} - 3f_{\nu\mu} \kappa^\mu = 0 \quad \text{for} \quad a \neq 0. \quad (3.18)$$

These relations are trivially satisfied for a Weyl vector field being “pure gauge”, i.e. implying $f_{\mu\nu} = 0$. This is identical with the condition (3.2) being satisfied, which may be written as

$$\kappa_\mu = -\partial_\mu \log \Phi^2. \quad (3.19)$$

The Weyl vector field in this broken Weyl theory is thus derivable from a potential given by the modulus of the scalar field. This is in direct analogy to the case of the Christoffel connection, $\bar{\Gamma}_{\mu\nu}{}^\rho = \{\frac{\rho}{\mu\nu}\}$, following from the relation $\bar{\nabla}_\rho g_{\mu\nu} = 0$ in (pseudo-)Riemannian geometry with $g_{\mu\nu} \in Gl(4, R)/SO(3, 1)$.

The field equations (3.7), finally, lead — together with (3.11) which implies $D_\rho R = 0$ for $D_\rho \Phi^2 = 0$ — to

$$\kappa_\mu = 0; \quad \text{i.e.} \quad \Phi^2 = \text{const.} \quad (3.20)$$

This shows that the Weyl space W_4 reduces completely to a pseudo-Riemannian space V_4 with the scalar field possessing a constant modulus. The value of this modulus can not be computed numerically. On the other hand, the value for Φ will determine the fermion and gauge boson masses appearing in the broken Weyl theory as well as in the nonlinearity contained in the ϕ -equation yielding the “Higgs dynamics” in the standard model. A fixing of an ungauged $D(1)$ degree of freedom is, indeed, implicit in the standard model. However, a relation of the type (3.2) for the $D(1)$ gauge symmetry breaking does not appear in the standard model since this would require to go beyond the flat space formulation of the conventional description. In the present context we have to investigate, in the V_4 limit given by (3.20), the appearance of Einstein’s equations for the metric coupled to the energy-momentum tensors of the now massive fermion and gauge boson fields and establish the fact that gravitation, as we know it from general relativity, is a natural part of the broken G -gauge dynamics described by \mathcal{L} .

A. Electromagnetism in the WEW Theory

In this subsection we first turn to the field equations for the electromagnetic fields $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the fields $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ following from Eqs. (3.8) and (3.9). The total \tilde{G} -curvature, written in Lie algebra valued form with $\mathcal{F}_{\mu\nu} = F_{\mu\nu}^a \frac{1}{2} \tau_a$, is [compare Eqs. (2.33) and (2.34)]

$$\mathcal{F}_{\mu\nu} + \frac{1}{2} \mathbf{1} B_{\mu\nu} = \partial_\mu \tilde{\Gamma}_\nu - \partial_\nu \tilde{\Gamma}_\mu + i\tilde{g} [\tilde{\Gamma}_\mu, \tilde{\Gamma}_\nu], \quad (3.21)$$

with $\tilde{\Gamma}_\mu$ as given by (B9). The definitions of the curvature components $F_{\mu\nu}^a$ and $B_{\mu\nu}$ imply that in a spherical basis we have, with Eqs. (2.73) and (2.77), the relations

$$F_{\mu\nu}^3 = \tilde{g}\tilde{g}_0^{-1} Z_{\mu\nu} + \tilde{g}'\tilde{g}_0^{-1} \frac{e}{\hbar c} F_{\mu\nu} - i\tilde{g} (W_\mu^\dagger W_\nu - W_\nu^\dagger W_\mu), \quad (3.22)$$

$$B_{\mu\nu} = -\tilde{g}'\tilde{g}_0^{-1} Z_{\mu\nu} + \tilde{g}\tilde{g}_0^{-1} \frac{e}{\hbar c} F_{\mu\nu}, \quad (3.23)$$

$$F_{\mu\nu}^- = (F_{\mu\nu}^+)^{\dagger} = \partial_\mu W_\nu - \partial_\nu W_\mu - i \left\{ \tilde{g} \cos \theta_W [W_\mu Z_\nu - W_\nu Z_\mu] + \tilde{e} \frac{e}{\hbar c} [W_\mu A_\nu - W_\nu A_\mu] \right\}, \quad (3.24)$$

with $F_{\mu\nu}^\pm = \frac{1}{\sqrt{2}}(F_{\mu\nu}^1 \pm iF_{\mu\nu}^2)$. In (3.21) $F_{\mu\nu}^3 \frac{1}{2}\tau_3 + \frac{1}{2}\mathbf{1}B_{\mu\nu}$ is that part which commutes with \hat{q} while $F_{\mu\nu}^\pm$, as defined by (3.24), denote the off diagonal parts which do not commute with \hat{q} .

We now rewrite the field equations (3.8) and (3.9) — the latter at first for $a = 3$ — using (3.22), (3.23) and the definition (2.66) of the electromagnetic current and find, with $\tilde{D}_\mu F_3^{\mu\rho} \equiv D_\mu F_3^{\mu\rho}$ and Eqs. (2.77) and (2.78),

$$\frac{e}{\hbar c} D_\mu F^{\mu\rho} = \tilde{\delta}^{-1} \tilde{e} j^{(e.m.)\rho} + i \tilde{e} D_\mu (W^{\dagger\mu} W^\rho - W^{\dagger\rho} W^\mu), \quad (3.25)$$

and, with $j_\rho^{(0)}$ as defined in (2.79),

$$D_\mu Z^{\mu\rho} = \tilde{\delta}^{-1} \tilde{g}_0 j^{(0)\rho} + i \tilde{e} \tilde{g} \tilde{g}'^{-1} D_\mu (W^{\dagger\mu} W^\rho - W^{\dagger\rho} W^\mu). \quad (3.26)$$

We continue to write here the covariant derivative as D_μ disregarding for the moment that $\kappa_\mu = 0$ according to (3.20) in the broken case since (3.25) and (3.26) are valid also in the Weyl symmetric theory discussed in Sect. II. The l.-h. side of (3.25) could also be written $D_\mu \bar{F}^{\mu\rho}$ with the electromagnetic field strengths $\bar{F}^{\mu\rho} = \partial^\mu \bar{A}^\rho - \partial^\rho \bar{A}^\mu$ of dimension $[L^{-2}]$. Eqs. (3.25), moreover, show that besides the electromagnetic source current $j_\rho^{(e.m.)}$ there contributes also a term on the r.-h. side which is bilinear in the W_μ -fields with W_μ and W_μ^\dagger being related to the charge changing weak processes. The same remark applies to the last term in (3.26) representing the contribution of the W_μ -fields to the neutral weak processes.

Furthermore, Eqs. (3.25) and (3.26) imply current conservation [compare (2.72)]

$$D^\rho j_\rho^{(e.m.)} \equiv \bar{\nabla}^\rho j_\rho^{(e.m.)} = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\rho} j_\rho^{(e.m.)}) = 0, \quad (3.27)$$

and, analogously, $D^\rho j_\rho^{(0)} = 0$ with the Weyl-covariant divergence of the W -terms and of the l.-h. sides of these equations yielding zero. Writing Eqs. (3.25) and (3.26) in the electromagnetic gauge (see Appendix B) it follows from (B5) – (B8) and (B17) that these equations are $U(1)_{e.m.}$ gauge invariant.

It may be worth while in this context to write down explicitly the $U(1)_{e.m.}$ gauge invariant source currents $\hat{j}_\rho^{(e.m.)}$ and $\hat{j}_\rho^{(0)}$ in the electromagnetic gauge [compare Eqs. (2.66), (2.79), (B3), (B15) and (B16)]

$$\begin{aligned} \hat{j}_\rho^{(e.m.)} &= -\hat{e}_L \gamma_\rho \hat{e}_L - \hat{e}_R \gamma_\rho \hat{e}_R \\ &= -(\hat{\varphi}_0)^{-2} \left\{ |\varphi_+|^2 (\bar{\nu}_L \gamma_\rho \nu_L) - |\varphi_+|^2 (\bar{e}_L \gamma_\rho e_L) + \varphi_+ \varphi_0^* (\bar{\nu}_L \gamma_\rho e_L) \right. \\ &\quad \left. + \varphi_+^* \varphi_0 (\bar{e}_L \gamma_\rho \nu_L) \right\} - \bar{\psi}_e \gamma_\rho \psi_e, \end{aligned} \quad (3.28)$$

and

$$\hat{j}_\rho^{(0)} = -\frac{1}{8}\tilde{g}\tilde{g}'(\hat{\varphi}_0)^2 \left(\hat{Z}_\rho + \hat{Z}_\rho^\dagger \right) + \frac{1}{2} \left(\hat{\nu}_L \gamma_\rho \hat{\nu}_L \right) - \frac{1}{2} \left(\hat{e}_L \gamma_\rho \hat{e}_L \right) - \sin^2 \theta_W \hat{j}_\rho^{(e.m.)}. \quad (3.29)$$

We, finally, determine the field equations for $F_{\mu\nu}^\pm$, i.e. for $a = 1, 2$ in (3.9). They read:

$$\tilde{\tilde{D}}_\mu (F^-)^{\mu\rho} \equiv D_\mu (F^-)^{\mu\rho} - i\tilde{g} W_\mu F_3^{\mu\rho} + i\tilde{g} A_\mu^3 (F^-)^{\mu\rho} = \tilde{\delta}^{-1} \tilde{g} j^{(1)\rho}, \quad (3.30)$$

$$\tilde{\tilde{D}}_\mu (F^+)^{\mu\rho} \equiv D_\mu (F^+)^{\mu\rho} + i\tilde{g} W_\mu^\dagger F_3^{\mu\rho} - i\tilde{g} A_\mu^3 (F^+)^{\mu\rho} = \tilde{\delta}^{-1} \tilde{g} j^{(1)\rho\dagger}, \quad (3.31)$$

where [compare (2.62) and (2.74)]

$$j_\rho^{(1)} = \frac{1}{\sqrt{2}} \left(j_{1\rho}^{(\phi)} + j_{1\rho}^{(\psi_L)} - i j_{2\rho}^{(\phi)} - i j_{2\rho}^{(\psi_L)} \right) \quad (3.32)$$

is the charge changing current. In (3.30) and (3.31) $F_3^{\mu\rho}$ and A_μ^3 may be replaced according to Eqs. (3.22) and (2.73) in order to yield field equations involving only the fields A_μ, Z_μ and W_μ, W_μ^\dagger . Written in the electromagnetic gauge Eqs. (3.30) and (3.31) are again $U(1)_{e.m.}$ gauge covariant. To establish this result one needs the formula

$$\left(\hat{F}_{\mu\nu}^\mp \right)' = e^{\mp i \frac{e}{\hbar c} \alpha(\phi', \phi)} \hat{F}_{\mu\nu}^\mp, \quad (3.33)$$

which is easily derivable from (3.24) with the help of (B5) – (B8). Moreover, one needs the following expression for the the current $\hat{j}_\rho^{(1)}$ evaluated at the origin in \tilde{G}/H . With the help of Eqs. (B3) and (B16) one finds

$$\hat{j}_\rho^{(1)} = -\frac{1}{4}\tilde{g}(\hat{\varphi}_0)^2 \hat{W}_\rho + \frac{1}{\sqrt{2}} \hat{e}_L \gamma_\rho \hat{\nu}_L. \quad (3.34)$$

From the form of (3.34) the transformation rule

$$\left(\hat{j}_\rho^{(1)} \right)' = e^{-i \frac{e}{\hbar c} \alpha(\phi', \phi)} \hat{j}_\rho^{(1)} \quad (3.35)$$

under residual $U(1)_{e.m.}$ gauge transformations is at once apparent as a consequence of (B6) and (B17), and correspondingly for $\hat{j}_\rho^{(1)\dagger}$ appearing in Eq. (3.31) after transformation to the electromagnetic gauge.

In order to gain information about the constant $\tilde{\delta}$ in Eqs. (3.25) and (3.26) and bring (3.25) — disregarding the W_μ -contributions for a moment — into the form of Maxwell's equations in electromagnetism, we first observe that each term in these equations has length dimension $[L^{-3}]$. Multiplying (3.25) by the charge e and introducing the fine-structure constant $\alpha_F = \tilde{e}^2/4\pi$ we can rewrite Eqs. (3.25) and (3.26) as

$$D_\mu F^{\mu\rho} = \frac{4\pi}{c} \mathbf{j}^{(e.m.)\rho} + i \frac{4\pi}{\tilde{e}} e D_\mu \left(W^{\dagger\mu} W^\rho - W^{\dagger\rho} W^\mu \right), \quad (3.36)$$

$$D_\mu Z^{\mu\rho} = \frac{1}{c} \mathbf{j}^{(0)\rho} + i \tilde{e} \tilde{g} \tilde{g}'^{-1} D_\mu \left(W^{\dagger\mu} W^\rho - W^{\dagger\rho} W^\mu \right), \quad (3.37)$$

by taking (compare (3.47) and (3.48) below)

$$\tilde{\delta} = \tilde{\delta}' l_\varphi^2 \quad \text{with} \quad \tilde{\delta}' = \frac{1}{\tilde{e}}, \quad (3.38)$$

Furthermore, we have introduced here the following current densities:

$$\mathbf{j}^{(e.m.)\rho} = e c l_\varphi^{-2} j^{(e.m.)\rho}; \quad \mathbf{j}^{(0)\rho} = \tilde{e} \tilde{g}_0 c l_\varphi^{-2} j^{(0)\rho}. \quad (3.39)$$

We have measured in (3.38) the constant $\tilde{\delta}$ of dimension $[L^2]$ in units of l_φ^2 defined in Eq. (3.47) and determined the numerical coefficient $\tilde{\delta}'$ in such a way that the first term on the r.-h. side of (3.36) has the conventional form known from electromagnetism. This fixed the numerical constant $\tilde{\delta}'$ to the value $\tilde{e}^{-1} = 1/\sqrt{4\pi\alpha_F}$.

B. Gravitation in the Broken WEW Theory

Einstein's equations for the metric follow from (3.10), as we will now show, with a total energy-momentum tensor $T_{\mu\nu}$ on the r.-h. side for all the interacting massive and massless fields involved. Clearly, $T_{\mu\nu}^{(f)} = 0$ as a consequence of (3.20). We first turn to the contributions of the \tilde{G} -gauge fields contained in $T_{\mu\nu}^{(B)} + T_{\mu\nu}^{(Fa)}$ on the r.-h. side of (3.10) and split this expression into the familiar electromagnetic contribution, a Z_μ -contribution, and W_μ -contributions in the following way:

$$T_{\mu\nu}^{(B)} + T_{\mu\nu}^{(Fa)} = T_{\mu\nu}^{(F)} + T_{\mu\nu}^{(Z)} + T_{\mu\nu}^{(W)} + T_{\mu\nu}^{(WW)}. \quad (3.40)$$

Using (3.38) we have here introduced the following energy-momentum tensors possessing the dimension $[L^{-2}]$:

$$T_{\mu\nu}^{(F)} = -\frac{\tilde{e}}{4\pi} \frac{1}{\hbar c} l_\varphi^2 \left[F_{\mu\sigma} F_\nu^\sigma - \frac{1}{4} g_{\mu\nu} F^{\rho\lambda} F_{\rho\lambda} \right] \quad (3.41)$$

for the electromagnetic fields, and

$$T_{\mu\nu}^{(Z)} = -\frac{1}{\tilde{e}} l_\varphi^2 \left[Z_{\mu\sigma} Z_\nu^\sigma - \frac{1}{4} g_{\mu\nu} Z^{\rho\lambda} Z_{\rho\lambda} \right], \quad (3.42)$$

$$T_{\mu\nu}^{(W)} = -\frac{1}{\tilde{e}} l_\varphi^2 \left[F_{\mu\sigma}^+ F_\nu^{-\sigma} + F_{\mu\sigma}^- F_\nu^{+\sigma} - \frac{1}{2} g_{\mu\nu} F^{+\rho\lambda} F_{\rho\lambda}^- \right], \quad (3.43)$$

for the Z - and W -fields, respectively. The last term, $T_{\mu\nu}^{(WW)}$, in (3.40) is a lengthy expression constructed with the fields $Z_{\mu\nu}$, $F_{\mu\nu}$ and W_μ^\dagger, W_ν containing terms of second and fourth order in the W -fields which we shall not write down explicitly. All terms in (3.40) are traceless, so that the contributions of the masses for the Z - and W -fields cannot come from these expressions but must be contained in the other contributions on the r.-h side of (3.10). In fact, it is the energy-momentum tensor of the ϕ -field, (3.12), which contains — besides the mass $\sqrt{a}m$ of the ϕ -field itself — the effects of the nonzero masses for the Z - and W -fields. In order to see this more clearly we consider the symmetry breaking relation (3.2) which implies, by taking the Weyl-covariant divergence,

$$D^\mu D_\mu \Phi^2 = 0. \quad (3.44)$$

Relating this to the field equation (3.4) for ϕ and its adjoint leads to the following result:

$$\left(\tilde{D}_\mu \phi \right)^\dagger \left(\tilde{D}^\mu \phi \right) = \frac{1}{6} R \phi^\dagger \phi + 4\beta \left(\phi^\dagger \phi \right)^2 + a \left[\frac{mc}{\hbar} \right]^2 \phi^\dagger \phi - \tilde{\gamma} \left[(\bar{\psi}_L \phi) \psi_R + \bar{\psi}_R (\phi^\dagger \psi_L) \right]. \quad (3.45)$$

Evaluating (3.45) at the origin $\hat{\phi}$ of \tilde{G}/H [compare Appendix A], i.e. considering (3.45) in the electromagnetic gauge, using moreover $D_\mu \hat{\phi}_0 = 0$ for $\hat{\phi}_0 \neq 0$, i.e. $\kappa_\mu = 0$, $\hat{\phi}_0 = \text{const}$ according to (3.20), one finds with the help of Eqs. (B3), (B18), (3.11) and (3.16)

$$\begin{aligned} \frac{1}{2} \tilde{g}^2 \hat{\phi}_0^2 \hat{W}_\mu^\dagger \hat{W}^\mu + \frac{1}{4} \tilde{g}_0^2 \hat{\phi}_0^2 \hat{Z}_\mu \hat{Z}^\mu &= \frac{1}{6} \bar{R} \hat{\phi}_0^2 + 4\beta \hat{\phi}_0^4 + a \left[\frac{mc}{\hbar} \right]^2 \hat{\phi}_0^2 - \tilde{\gamma} \hat{\phi}_0 \hat{\psi}_e \hat{\psi}_e \\ &= 4\beta \hat{\phi}_0^4 + \left[\frac{mc}{\hbar} \right]^2 (a - \hat{\phi}_0^2) \hat{\phi}_0^2 - \tilde{\gamma} \hat{\phi}_0 \hat{\psi}_e \hat{\psi}_e. \end{aligned} \quad (3.46)$$

This is an interesting relation between the mass terms for the various fields involved in the theory and the nonlinear self-coupling term of the scalar field.

We first observe that each term in (3.46) has dimension $[L^{-2}]$ and that the unit of lengths in which every quantity with a length dimension is to be measured is given by the length

$$l_\varphi = \frac{\hbar}{mc} \quad (3.47)$$

appearing on the r.-h. side of (3.46). This length was introduced by the Weyl-symmetry breaking Lagrangean \mathcal{L}_B defined in (3.1). Indeed, for $a = 1$ the mass of the scalar field is $m_\phi \equiv m$ and the corresponding Compton wave length is l_φ . This length will from now on be adopted as the unit of lengths. This choice implies that all quantities with a length dimension have to be measured in units of l_φ as we already did for the constant $\tilde{\delta}$ in (3.38). For the constants β and $\tilde{\gamma}$ this means that

$$\beta = \beta' l_\varphi^{-2}, \quad \tilde{\gamma} = -\tilde{\gamma}' l_\varphi^{-1}, \quad (3.48)$$

with the primed quantities being numerical constants. The minus sign in the second equation is adopted here in order to obtain the correct sign for the electron mass in the last term of (3.46) which is thus given by

$$-\tilde{\gamma} \hat{\phi}_0 = \tilde{\gamma}' \hat{\phi}_0 \frac{mc}{\hbar} = \frac{m_e c}{\hbar}. \quad (3.49)$$

The same argument applies to the Z - and W -boson fields of length dimension $[L^{-1}]$ and the corresponding masses. We thus identify the masses of the charged ($q = \mp 1e$) and neutral ($q = 0$) boson fields as well as the electron mass m_e by the equations:

$$2m_W^2 = \frac{1}{2} \tilde{g}^2 \hat{\phi}_0^2 m^2; \quad m_Z^2 = \frac{1}{4} \tilde{g}_0^2 \hat{\phi}_0^2 m^2; \quad m_e = \tilde{\gamma}' \hat{\phi}_0 m, \quad (3.50)$$

implying the relation

$$m_Z = \frac{m_W}{\cos \theta_W} \quad (3.51)$$

between the Z - and the W -masses, which is well-known from the standard model. Below we shall sometimes denote by \tilde{m}_W , \tilde{m}_Z and \tilde{m}_e the dimensionless quantities

$$\tilde{m}_W = \frac{m_W}{m} = \frac{1}{2} \tilde{g} \hat{\phi}_0; \quad \tilde{m}_Z = \frac{m_Z}{m} = \frac{1}{2} \tilde{g}_0 \hat{\phi}_0; \quad \tilde{m}_e = \frac{m_e}{m} = \tilde{\gamma}' \hat{\phi}_0. \quad (3.52)$$

We now focus the attention on the energy-momentum tensor $\Theta_{\mu\nu}^{(\phi)'}$ for the field ϕ which was defined in (3.12), reading with $a = 1$

$$\Theta_{\mu\nu}^{(\phi)'} = \frac{1}{2} \left[\left(\tilde{D}_\mu \phi \right)^\dagger \left(\tilde{D}_\nu \phi \right) + \left(\tilde{D}_\nu \phi \right)^\dagger \left(\tilde{D}_\mu \phi \right) \right] - g_{\mu\nu} \left\{ \frac{1}{2} \left(\tilde{D}^\rho \phi \right)^\dagger \left(\tilde{D}_\rho \phi \right) - \beta \left(\phi^\dagger \phi \right)^2 - \frac{1}{2} \left[\frac{mc}{\hbar} \right]^2 \phi^\dagger \phi \right\}. \quad (3.53)$$

Using now the relation (3.45) for $a = 1$ in order to eliminate the $(\phi^\dagger \phi)^2$ -coupling term proportional to β in (3.53) one obtains, considering also (3.10) again,

$$\Theta_{\mu\nu}^{(\phi)'} = \frac{1}{2} \left[\left(\tilde{D}_\mu \phi \right)^\dagger \left(\tilde{D}_\nu \phi \right) + \left(\tilde{D}_\nu \phi \right)^\dagger \left(\tilde{D}_\mu \phi \right) \right] - \frac{1}{4} g_{\mu\nu} \left\{ \left(\tilde{D}^\rho \phi \right)^\dagger \left(\tilde{D}_\rho \phi \right) - \left[\frac{mc}{\hbar} \right]^2 \left(\Phi^2 + 1 \right) \Phi^2 - \tilde{\gamma} \left[(\psi_L \phi) \psi_R + \bar{\psi}_R (\phi^\dagger \psi_L) \right] \right\}. \quad (3.54)$$

Evaluating this in the electromagnetic gauge yields with Eqs. (B3), (3.49), (3.50), and (3.52), and with $\hat{Z}_\mu^\dagger = \hat{Z}_\mu$, the result

$$\hat{\Theta}_{\mu\nu}^{(\phi)'} = \tilde{m}_W^2 \left(\hat{W}_\mu^\dagger \hat{W}_\nu + \hat{W}_\nu^\dagger \hat{W}_\mu \right) + \tilde{m}_Z^2 \hat{Z}_\mu \hat{Z}_\nu - \frac{1}{4} g_{\mu\nu} \left\{ 2 \tilde{m}_W^2 \hat{W}_\rho^\dagger \hat{W}^\rho + \tilde{m}_Z^2 \hat{Z}_\rho \hat{Z}^\rho - \left[\frac{mc}{\hbar} \right]^2 \left(\hat{\varphi}_0^2 + 1 \right) \hat{\varphi}_0^2 + \frac{m_e c}{\hbar} \hat{\psi}_e \hat{\psi}_e \right\}, \quad (3.55)$$

showing that the boson and fermion mass terms appear in the total energy-momentum tensor through the tensor $\hat{\Theta}_{\mu\nu}^{(\phi)'}$.

The r.-h. side of (3.10), i.e. the source term of the field equations for the metric, when evaluated in the electromagnetic gauge and for $\kappa_\mu = 0$, $\hat{\varphi}_0 = \text{const}$ now reads

$$\hat{T}_{\mu\nu} = \left\{ \hat{\Theta}_{\mu\nu}^{(\phi)'} + \hat{T}_{\mu\nu}^{(\psi_L)} + \hat{T}_{\mu\nu}^{(\psi_R)} + g_{\mu\nu} \frac{m_e c}{\hbar} \hat{\psi}_e \hat{\psi}_e + \hat{T}_{\mu\nu}^{(F)} + \hat{T}_{\mu\nu}^{(Z)} + \hat{T}_{\mu\nu}^{(W)} + \hat{T}_{\mu\nu}^{(WW)} \right\}_{\kappa_\mu=0, \hat{\varphi}_0=\text{const}}, \quad (3.56)$$

where $\hat{\Theta}_{\mu\nu}^{(\phi)'}$ is given by (3.55), and (B18) has been used for the Yukawa term. After Weyl-symmetry breaking according to (3.1), (3.2) yielding (3.20) we thus have to evaluate the r.-h. side of (3.56) for a vanishing Weyl vector field and for $\hat{\varphi}_0 = \text{const}$, i.e. in a V_4 , which is indicated by the suffix on the curly brackets. Moreover, we use the length l_φ as a universal unit. By construction $\hat{T}_{\mu\nu}$ satisfies the usual conservation relations $\bar{\nabla}^\mu \hat{T}_{\mu\nu} = 0$ due to Eqs. (3.18) – (3.20).

The l.-h side of (3.10) for $R = \bar{R} = \text{const}$ according to Eqs. (3.11) and (3.20), and with $\tilde{\alpha} = 0$ — remembering that $\tilde{\alpha}$ was introduced in $\tilde{\mathcal{L}}_{W_4}$, as discussed in I, to yield a nontrivial dynamics for the Weyl vector fields which now vanish — is given in the electromagnetic gauge and in the V_4 limit, taking moreover $a = 1$ (see above), by

$$\frac{1}{6} \left(\hat{\varphi}_0^2 + 1 \right) \left[\bar{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \bar{R} \right]. \quad (3.57)$$

This together with (3.56) yields, finally, a set of field equations for the metric of the form

$$\bar{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \bar{R} = \frac{1}{\frac{1}{6}[\hat{\varphi}_0^2 + 1]K} \hat{T}_{\mu\nu}'. \quad (3.58)$$

Here appears the constant K in the denominator on the r.-h. side since we want to measure the total energy-momentum tensor ultimately in the conventional units of $[Energy/L^3]$ while $\hat{T}_{\mu\nu}$ in (3.56) has dimension $[L^{-2}]$. This we indicate by a prime on the total source tensor which is given by $\hat{T}_{\mu\nu}' = K \hat{T}_{\mu\nu}$. Eqs. (3.58) are identical with Einstein's field equations in general relativity provided we are entitled to make the identification

$$\kappa_E = \frac{1}{\frac{1}{6}[\hat{\varphi}_0^2 + 1]K}, \quad (3.59)$$

where κ_E is Einstein's gravitational constant, $\kappa_E = 8\pi N/c^4 = 2.076 \cdot 10^{-48} g^{-1} cm^{-1} sec^2$, and N is Newton's constant. Of course, in the framework adopted here also κ_E is to be expressed in the proper units related to l_φ as the chosen intrinsic fundamental length unit replacing the *cgs*-units conventionally chosen for κ_E yielding the quoted numerical value of this constant. (For a general discussion on the transformation of the units for mass, length and time we refer to Nariai and Ueno [8] and Dicke [9].) Eq. (3.59) implies that the over-all size of Einstein's gravitational constant and its dimension is determined by the constant K^{-1} with $\hat{\varphi}_0$, representing an elementary mass ratio if the coupling constants \tilde{g} , \tilde{g}_0 or $\tilde{\gamma}'$ in (3.52) were known, leading to a correction in the relation between κ_E and K as expressed by (3.59). The squared modulus of the scalar field — being a constant after Weyl-symmetry breaking — enters the gravitational constant κ_E in a manner reminiscent of the Brans-Dicke scalar-tensor theory of gravitation [3] although, as mentioned, the strength of the gravitational coupling is determined in the presented broken Weyl theory essentially by K^{-1} having the dimension $[L/Energy]$. For a more detailed investigation of this point see, however, Subsection D below. Furthermore, it is easy to show that taking the trace on both sides of (3.58) yields an identity after use of (3.11) has been made.

In the described situation where one considers the appearance of the unit of length as originating from a $D(1)$ - or Weyl-symmetry breaking in a theory containing a universal scalar quantum field — relating the established length unit to the mass of this field — the quantities \hbar and c as well as the fine-structure constant $\alpha_F = \tilde{e}^2/4\pi$ are regarded as universal constants being by definition unrelated to the appearance of the length l_φ . Summarizing we may say that the scalar field in its $U(1)_{e.m.}$ gauge invariant form $\hat{\phi} = \begin{pmatrix} 0 \\ \hat{\varphi}_0(x) \end{pmatrix}$, with $\hat{\varphi}_0(x)$ being a constant, $\hat{\varphi}_0$, after $D(1)$ -symmetry breaking, determines according to Eqs. (3.50) and (3.52) not only the Z - and W -boson masses as well as the electron mass in a way described as the “Higgs phenomenon” in the standard electroweak model — which, in fact, is just a choice of gauge called here the electromagnetic or nonlinear gauge — but affects also the gravitational coupling constant in a Brans-Dicke-like manner with $\hat{\Phi}^2 = \hat{\varphi}_0^2$ playing the role of the real scalar field.

C. The Field Equations for the Scalar Field

It is a surprising fact that the nonlinear ϕ^4 -coupling term proportional to the constant β could be eliminated from the energy-momentum tensor $\Theta_{\mu\nu}^{(\phi) \prime}$ due to Eq. (3.45) following

from the Weyl-symmetry breaking relation $D_\mu \Phi^2 = 0$. In the electromagnetic gauge Eq. (3.45) took the form of the first equation in (3.46) which then led, with $a = 1$ and Eqs. (3.48) – (3.52), to $\hat{\Theta}_{\mu\nu}^{(\phi) \prime}$ given in (3.55).

In concluding this section let us now finally study the field equation (3.4) for ϕ for the case $a = 1$, and $\kappa_\mu = 0$, $\hat{\varphi}_0 = \text{const}$ according to (3.20), to see the influence of the ϕ^4 -coupling on the dynamics of the gauge and fermion fields in the nonlinear, i.e. electromagnetic, gauge. To this end one has to compute $\hat{\tilde{D}}^\mu \hat{\tilde{D}}_\mu \hat{\phi}$ using (B1) and (B3). It is then easy to show that (3.4) is, in the electromagnetic gauge, equivalent to the following $U(1)_{e.m.}$ gauge covariant equations:

$$i \frac{\tilde{g}}{\sqrt{2}} \hat{\varphi}_0 \left[\bar{\nabla}^\mu \hat{W}_\mu + i \tilde{e} \hat{A}^\mu \hat{W}_\mu - i \tilde{g}_0 \hat{Z}^\mu \hat{W}_\mu \right] + 2 \tilde{\gamma}' \frac{mc}{\hbar} \hat{e}_R \hat{\nu}_L = 0, \quad (3.60)$$

$$i \frac{\tilde{g}}{\sqrt{2}} \hat{\varphi}_0 \left[\bar{\nabla}^\mu \hat{W}_\mu^\dagger - i \tilde{e} \hat{A}^\mu \hat{W}_\mu^\dagger + i \tilde{g}_0 \hat{Z}^\mu \hat{W}_\mu^\dagger \right] - 2 \tilde{\gamma}' \frac{mc}{\hbar} \hat{\nu}_L \hat{e}_R = 0, \quad (3.61)$$

$$- \left[2 \tilde{m}_W^2 \hat{W}^{\dagger\mu} \hat{W}_\mu + \tilde{m}_Z^2 \hat{Z}^\mu \hat{Z}_\mu \right] + \frac{1}{6} \bar{R} \hat{\varphi}_0^2 + 4 \beta \hat{\varphi}_0^4 + \left[\frac{mc}{\hbar} \right]^2 \hat{\varphi}_0^2 + \frac{m_e c}{\hbar} \hat{\psi}_e \hat{\psi}_e = 0, \quad (3.62)$$

$$\bar{\nabla}^\mu \hat{Z}_\mu = 2 \tilde{\gamma}' \frac{1}{\tilde{g}_0 \hat{\varphi}_0} \frac{mc}{\hbar} i \left(\hat{e}_L \hat{e}_R - \hat{e}_R \hat{e}_L \right). \quad (3.63)$$

Here (3.60) corresponds to the upper components in (3.4) when evaluated for $\hat{\phi} = \begin{pmatrix} 0 \\ \hat{\varphi}_0 \end{pmatrix}$, Eq. (3.61) is the adjoint equation of (3.60), while (3.62) and (3.63) are the real and imaginary parts of the lower components in (3.4), respectively. For comparison with (3.46) Eq. (3.62) was multiplied by $\hat{\varphi}_0$. The constant β only enters Eq. (3.62) which is seen to be identical to the first equation of (3.46) [prior to the use of (3.11) and (3.52)] which was derived above. Hence the only equation containing the term proportional to β is, in fact, the algebraic equation Eq. (3.62), and this equation was used above to eliminate the β -contribution from the source terms in Einstein's equations. The elimination of the β -term is a particular consequence of the Weyl-symmetry breaking by Eq. (3.2) in this Weyl-electroweak theory. Stated physically one may say: The energy represented by the term proportional to $\beta(\phi^\dagger \phi)^2$ in Eq. (3.53) may be reexpressed by the mass terms appearing in Eq. (3.62), using also (3.11), so that the constant β disappears from the final equations. The field equation for ϕ is thus, finally, turned into the set of linear differential equations (3.60), (3.61) and (3.63).

D. Determination of the Parameters of the Theory

The following parameters appearing in the Lagrangean (3.3) have already been fixed so far: $a = 1$ in (3.1); $\tilde{\alpha} = 0$ in $\tilde{\mathcal{L}}_{W_4}$ [compare (2.29)], and $\tilde{\delta}' = 1/\tilde{e}$ with $\tilde{e} = \sqrt{4\pi\alpha_F} = 0.30282$ in (3.38). Moreover, β disappeared from the dynamics after the Weyl-symmetry breaking as was shown in the last subsection. The remaining parameters to be determined are the following six quantities: The constants \tilde{g} and \tilde{g}' together with the Weinberg angle θ_W [compare (2.77)]; the constants $\hat{\varphi}_0$ and K ; the Yukawa coupling constant $\tilde{\gamma}'$ and, last not least, the universal length unit $l_\varphi = \hbar/mc$ or rather the reference mass $m = m_\phi$. Besides the

value for \tilde{e} already quoted (using $\alpha_F^{-1} = 137.04$) we have the following five experimental data at our disposal: $m_e = 0.510999 \text{ MeV}/c^2$, $m_Z = 91.187 \text{ GeV}/c^2$, $m_W = 80.41 \text{ GeV}/c^2$, $G_F = 1.16639 \cdot 10^{-5} \text{ GeV}^{-2}$ and $\kappa_E = 2.076 \cdot 10^{-48} \text{ g}^{-1} \text{ cm}^{-1} \text{ sec}^2$.

Unfortunately it is not possible to decide uniquely what the actual length scale l_φ is which is to be adopted as a universal unit in the theory. We shall investigate two conceivable possibilities in somewhat greater detail: (a) the mass of the ϕ -field is identical with the Z -boson mass, i.e. $m = m^{(a)} = m_Z$, with $l_\varphi = l_\varphi^{(a)} = \hbar/m_Z c = 0.2164 \cdot 10^{-15} \text{ cm}$, and (b) the mass of the ϕ -field is identical to the electron mass, i.e. $m = m^{(b)} = m_e$, with $l_\varphi = l_\varphi^{(b)} = \hbar/m_e c = 0.38610 \cdot 10^{-10} \text{ cm}$. As a third possibility we only mention briefly the case when $l_\varphi = l_\varphi^{(c)} = 1 \text{ cm}$ corresponding to $m = m^{(c)} = 0.1973 \cdot 10^{-4} \text{ eV}/c^2$. This possibility could be of interest in connection with a very small but nonzero neutrino mass of order $m^{(c)}$ as the lower edge of the fermion mass spectrum. Of course, this last choice, $l_\varphi = 1 \text{ cm}$, is completely ad hoc being included here only as an orientation.

For case (a) we find the following numerical values: $\cos \theta_W = 0.8818$, $|\tilde{g}| = 0.65316$ [from G_F], $\tilde{g}' = 0.34341$ and thus $|\tilde{g}_0| = 0.73794$; $\hat{\varphi}_0^2 = 7.345$ and $|\tilde{\gamma}'| = 0.2067 \cdot 10^{-5}$ [from $\hat{\varphi}_0 = (m_e/m_Z)\tilde{\gamma}'^{-1}$]. For $\hat{\varphi}_0 > 0$ the constants \tilde{g} , \tilde{g}_0 , and $\tilde{\gamma}'$ must be positive, i.e. the absolute signs in the quoted results are unnecessary. Finally one has $K = 0.7190/\kappa_E$, i.e. K is, for the case (a), essentially the inverse Einstein constant.

For the case (b) the constants \tilde{g} , \tilde{g}' , \tilde{g}_0 and $\tilde{\gamma}'$ are the same as for the case (a) given above. However, now we have $\hat{\varphi}_0^2 = 2.339 \cdot 10^{11}$ due to $\hat{\varphi}_0 = \tilde{\gamma}'^{-1}$. This leads, finally, to $K = 2.565 \cdot 10^{-11}/\kappa_E$ being a factor of the order of $(m_e/m_Z)^2$ smaller than in case (a). From this it is apparent that the relative contributions of $\hat{\varphi}_0^2$ and K in the relation (3.59) for κ_E depends strongly on the unit of lengths adopted.

In concluding this subsection we remark that the question of the size of the ‘‘Higgs mass’’ in the conventional formulation of the standard model has turned in the present broken Weyl-electroweak theory into the question of the relative contributions of $\hat{\varphi}_0^2$ and K to Einstein’s gravitational constant and, correspondingly, into the question of the actual size of the true elementary length scale to be adopted in the theory. We, moreover, mention that in our determination of the free parameters of the theory we used the observed electron, Z_0 - and W^\pm -boson masses disregarding radiative corrections.

The strength of the gravitational interaction is usually characterized by the Planck length $l_{Planck} = (N\hbar/c^3)^{\frac{1}{2}} = 1.616 \cdot 10^{-33} \text{ cm}$. With this Einstein’s gravitational constant κ_E may be written as $\kappa_E \cdot \hbar c = 8\pi l_{Planck}^2 = 65.64 \cdot 10^{-66} \text{ cm}^2$. However, writing finally Einstein’s field equations (3.58) relating the contracted space-time curvature, i.e. the Einstein tensor $G_{\mu\nu}$, to the distribution of energy and momentum in a form independent of a particular choice of a length unit yields

$$G_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\bar{R} = \frac{6}{[\hat{\varphi}_0^2 + 1]}T_{\mu\nu} \quad (3.64)$$

where $G_{\mu\nu}$ and $T_{\mu\nu}$ are both to be measured in the same units of an inverse length squared. The gravitational coupling in dimensionless form is characterized in Eqs. (3.64) by the constant $6/[\hat{\varphi}_0^2 + 1]$. For a massless world, i.e. for $\hat{\varphi}_0 = 0$ in Eqs. (3.52), this dimensionless coupling constant would at most be 6; for the case (a) above it would be 0.719 — i.e. of the order of unity as mentioned — and for the case (b) it would be $2.565 \cdot 10^{-11}$.

IV. DISCUSSION

We investigated in this paper the semi-classical theory of a scalar-isospinor field ϕ coupled to the chiral fermion fields ψ_L and ψ_R in the presence of the gauge fields κ_μ (Weyl vector fields) for the dilatation group $D(1)$, and the gauge fields A_μ, Z_μ, W_μ and W_μ^\dagger for the electroweak gauge group $\tilde{G} = SU(2)_W \times U(1)_Y$. The dynamics of this originally massless and scaleless theory was formulated in a Weyl space W_4 characterized by a family of metrics $g_{\mu\nu}$ and associated Weyl vector fields κ_μ , both determined only up to Weyl transformations (2.3) and (2.4) corresponding to conformal rescalings of the metric and the related transformations of the Weyl vector fields, respectively. The gauge structure of the original Weyl-electroweak theory (WEW theory) was given by the group $G = SO(3, 1) \otimes D(1) \otimes \tilde{G}$.

In order to investigate the appearance of nonzero masses and establish a scale of lengths, l_φ , in the theory which is associated with the squared modulus, $\Phi^2 = \phi^\dagger \phi$, of Weyl weight -1 of the scalar field and, furthermore, derive field equations of Einstein's type for the metric, we broke the Weyl-symmetry explicitly by a term in the Lagrangean involving the scalar curvature R of the W_4 and a mass term for the scalar field. The idea here is to establish an intrinsic length scale in an originally massless and scaleless Weyl-symmetric theory by attributing this nonzero mass and corresponding finite length unit to the scalar field ϕ . Then we studied how nonzero masses for the various other interacting fields appear on the scene within the framework of a broken gauge theory containing as a subsymmetry the electroweak gauge symmetry which is known to contain many features in accord with observation. After the Weyl-symmetry breaking we finally obtain a $U(1)_{e.m.}$ gauge covariant theory formulated in a Riemannian space V_4 . The reduction of the Weyl geometry to a Riemannian geometry for the underlying space-time is governed by a true symmetry breaking relation, $D_\mu \Phi^2 = 0$, implying that the $D(1)$ gauge field κ_μ is "pure gauge" with the associated length curvature $f_{\mu\nu}$ being zero and the gauge symmetry with group G reducing to a gauge symmetry with the subgroup $G' = SO(3, 1) \otimes \tilde{G}$. This is different from the so-called spontaneous symmetry breaking in the electroweak sector of the theory which is better described as a choice of gauge by singling out a particular point $\hat{\phi}$ as origin in the coset space \tilde{G}/H , with $\hat{\phi}$ being invariant under $H \equiv U(1)_{e.m.}$, where \tilde{G}/H is isomorphic to the scalar field ϕ (see Appendix A).

The transformation $\phi \longrightarrow \hat{\phi}$ is a gauge transformation which reshuffles the fields by putting the theory in a form possessing a residual $U(1)_{e.m.}$ gauge freedom and exhibiting the appearance of mass terms for the \hat{Z}_μ -field, the \hat{W}_μ - and \hat{W}_μ^\dagger -fields, and the electron field $\hat{\psi}_e$ without, however, reducing the connection and covariant derivatives from a Lie \tilde{G} -valued form to a form characterized by a corresponding expression associated with a subgroup of \tilde{G} . This is contrary to the situation for the $D(1)$ or Weyl-symmetry breaking described in this paper which, indeed, is a true symmetry reduction $G \longrightarrow G'$ in the sense of the theorem quoted at the end of Sec. II leading to the appearance of the length scale l_φ in the theory freezing at the same time the squared modulus $\hat{\Phi}^2$ of the scalar field to the constant $\hat{\varphi}_0^2$. Now the question arises: What is the true nature of this scalar "field" $\hat{\varphi}_0$ which enters the Z_0 - and W^\pm -boson masses relating them and the electron mass to the established length scale and, furthermore, enters the gravitational constant in Einstein's field equations for

the metric in the V_4 limit in a manner comparable to a Brans-Dicke field $\hat{\varphi}_0^2$. We try to answer this question in the following way: The ϕ -field, as it appears in the broken Weyl theory, is a *vehicle for symmetry breaking*. ϕ is not a matter field of the usual type which would also possess a particle interpretation in a fully quantized theory. For this reason it is very unlikely that this field, being in the V_4 limit reduced to a constant responsible for the mass generation of the gauge boson and charged fermion fields, would actually show up as a particle in high energy processes. It is a field necessary to establish a scale in a theory. Now the next question arises: What is the actual size of this scale? Of course, here we have to rely on observation. Since the measured masses m_Z and m_W are of the order of $100\text{ GeV}/c^2$ it is suggestive to assume that the mass scale established by the ϕ -field after Weyl-symmetry breaking is of this same order. The corresponding length l_φ would thus be of the order of $l_\varphi \approx 0.2 \cdot 10^{-15}\text{ cm}$ [case (a) in Subsection III D]. However, this identification, although reasonable, is not compelling. Regardless of whether one fixes the mass m at the scale of $100\text{ GeV}/c^2$ — case (a) above — or identifies this ϕ -mass with the much smaller electron mass — case (b) above — the huge difference of the observed masses for the Z_0 - and W^\pm -bosons, on the one hand, and the electron mass, on the other hand, is mainly due to the Yukawa coupling constant $\tilde{\gamma}'$ [compare Eqs. (3.52)]. A point of particular interest in the presented broken Weyl theory, however, is that the special choice of the unit of lengths also affects the dimensionless coupling constant appearing in the field equations (3.64) for gravity.

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APPENDIX A: COSET REPRESENTATION OF ϕ

The coset representation of ϕ is related to the proper disentanglement of the various $U(1)$ phase groups involved. The field ϕ transforms under $G = SO(3, 1) \otimes D(1) \otimes \tilde{G}$ corresponding to a representation with spin zero, Weyl weight $w(\phi) = -\frac{1}{2}$, isospin $I = \frac{1}{2}$, and hypercharge $Y = \frac{1}{2}$. The $D(1)$ factor of G affects the modulus $\Phi = \sqrt{\phi^\dagger \phi}$, while the electroweak gauge group

$$\tilde{G} = SU(2)_W \times U(1)_Y \quad (\text{A1})$$

determines the orientation of the isospin degrees of freedom and the $U(1)_Y$ phase. We denote by $U(\bar{g})$, with $\bar{g} \in \tilde{G}$, the 2×2 representation of \tilde{G} operating on ϕ . We parametrize the elements of \tilde{G} by $\bar{g} = \bar{g}(b_a, \beta)$ with b_a ; $a = 1, 2, 3$, yielding a parametrization of $SU(2)_W$, and with β parametrizing the hypercharge transformations. (The angle β should not be confused with the constant β used in Eq. (2.29) and in the main text.) Thus the $U(1)_Y$ transformations are given by

$$U(\bar{g}(0, \beta)) = \begin{pmatrix} e^{i\bar{g}'Y\beta} & 0 \\ 0 & e^{i\bar{g}''Y\beta} \end{pmatrix} \quad (\text{A2})$$

with $Y = \frac{1}{2}$. Mathematically speaking, the transformations (A2) are transformations of $U(2)$ which may be decomposed into the direct product

$$U(\bar{g}(0, \beta)) = U(1)_+ \otimes U(1)_- , \quad (\text{A3})$$

where the groups $U(1)_\pm$ are generated by $\frac{1}{2}(1 \pm \tau_3)$, respectively, i.e. by the electromagnetic charge $\hat{q} = \frac{1}{2}(1 + \tau_3)$ [compare Eq. (2.68)] and by

$$\hat{q}_0 = \frac{1}{2}(1 - \tau_3). \quad (\text{A4})$$

The decomposition of the original weak hypercharge and isospin transformations into \hat{q} and \hat{q}_0 contributions (of which, as shown below, only the \hat{q} contributions survive) corresponds to physically measurable situations yielding the coupling to the A_ρ -fields (electromagnetic effects) and the coupling to the Z_ρ -fields (weak neutral effects).

We now like to introduce a representation of ϕ which is characterized in terms of the cosets \tilde{G}/H , where H is the *electromagnetic* subgroup of \tilde{G} , i.e. $H = U(1)_{e.m.} = U(1)_+ \subset \tilde{G}$. We thus write

$$\phi = U(\bar{g}_\phi) \hat{\phi} \quad \text{with} \quad \hat{q} \hat{\phi} = 0. \quad (\text{A5})$$

Here $U(\bar{g}_\phi)$ is an element of \tilde{G}/H parametrized by ϕ , and

$$\hat{\phi} = \begin{pmatrix} 0 \\ \hat{\varphi}_0 \end{pmatrix}, \quad (\text{A6})$$

with $\hat{\varphi}_0$ being a real field, denotes the origin of the coset space. $\hat{\phi}$ is invariant under the electromagnetic gauge group $U(1)_{e.m.} = U(1)_+$ [the stability group H] with transformations $h \in H$ given by

$$U(h(\alpha)) = e^{-i\frac{e}{\hbar c}\hat{q}\alpha} = \begin{pmatrix} e^{-i\frac{e}{\hbar c}\alpha} & 0 \\ 0 & 1 \end{pmatrix}, \quad (\text{A7})$$

where the minus sign in the exponential is adopted for conventional reasons. Due to the splitting (A3) of the hypercharge transformations and the invariance of $\hat{\phi}$ by the contributions $U(1)_+$ generated by the charge \hat{q} , the transformation $U(\bar{g}_\phi)$ — which could be called a “boost” generating ϕ from the fixed state $\hat{\phi}$ — is seen to be given by the following element of $SU(2)_W \otimes U(1)_-$:

$$U(\bar{g}_\phi) = U(\bar{g}(b_a(\phi))) e^{\frac{i}{2}\tilde{g}'\hat{q}_o\beta(\phi)}, \quad (\text{A8})$$

with \hat{q}_o as given by (A4). Here the first factor on the r.-h. side is an element of $SU(2)_W$ parametrized by $b_a(\phi)$, and the second factor is the transformation

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i}{2}\tilde{g}'\beta(\phi)} \end{pmatrix} \in U(1)_- \quad (\text{A9})$$

with hypercharge phase angle $\beta(\phi)$. It is easy to show that one can express the r.-h. side of (A8) in terms of the components of ϕ in the following manner [compare (2.23)]:

$$U(\bar{g}_\phi) = \frac{1}{\hat{\varphi}_0} \begin{pmatrix} \varphi_0^* e^{\frac{i}{2}\tilde{g}'\beta(\phi)} & \varphi_+ \\ -\varphi_+^* e^{\frac{i}{2}\tilde{g}'\beta(\phi)} & \varphi_0 \end{pmatrix}, \quad (\text{A10})$$

with $\hat{\varphi}_0 = \sqrt{\phi^\dagger \phi} = \sqrt{|\varphi_+|^2 + |\varphi_0|^2}$ expressing the invariance of the modulus Φ of ϕ under “boosts” parametrized in terms of \tilde{G}/H . Identifying ϕ with the coset $U(\phi) \cdot H \equiv U(\bar{g}_\phi) \cdot H$ we may now take the simpler matrix $U(\phi)$ defined by

$$U(\phi) = \frac{1}{\hat{\varphi}_0} \begin{pmatrix} \varphi_0^* & \varphi_+ \\ -\varphi_+^* & \varphi_0 \end{pmatrix}, \quad \text{with} \quad \det U(\phi) = 1 \quad (\text{A11})$$

as a coset representative instead of $U(\bar{g}_\phi)$ and write Eq. (A5) as

$$\phi = U(\phi) \hat{\phi}. \quad (\text{A12})$$

The \tilde{G} -transformation of ϕ or, more exactly, the gauge transformation $U(\bar{g}(x))$ representing a change of section on the bundle E [see (2.25)] given by (the argument x is suppressed)

$$\phi' = U(\bar{g}) \phi, \quad (\text{A13})$$

together with the coset representation (A12) of ϕ as well as of ϕ' associated with the stability group $H = U(1)_{e.m.}$ now yields the following decomposition of an arbitrary transformation $U(\bar{g})$ into boosts parametrized by $\phi = \phi(x)$ and $\phi' = \phi'(x)$, respectively, and a stability

group transformation characterized by an angle α depending on $\phi(x)$ and on $\bar{g} = \bar{g}(x)$ which is written for short as $\alpha(\phi', \phi)$:

$$U(\bar{g}) = U(\phi') e^{-i\frac{e}{\hbar c}\hat{q}\alpha(\phi', \phi)} U^{-1}(\phi). \quad (\text{A14})$$

Here the subgroup transformation

$$U(h(\phi', \phi)) = e^{-i\frac{e}{\hbar c}\hat{q}\alpha(\phi', \phi)} = \begin{pmatrix} e^{-i\frac{e}{\hbar c}\alpha(\phi', \phi)} & 0 \\ 0 & 1 \end{pmatrix} \quad (\text{A15})$$

could be called the “Wigner rotation” for the electroweak theory or the “little group transformation” at the origin $\hat{\phi}$ in the coset space \tilde{G}/H which is associated with the transformation $U(\bar{g}) = U(\bar{g}(b_a, \beta))$ sending ϕ into ϕ' . The transformation (A15) with angle $\alpha(\phi', \phi)$ is in similar contexts usually called the *nonlinear realization* of a gauge transformation of the group \tilde{G} on the stability subgroup H of G [6]. In the standard model, however, this terminology is not used and one speaks instead of a symmetry breaking by the vacuum expectation value of the scalar field ϕ having the form (A6). In the present case, with \tilde{G} possessing the product structure (A1), one finds for the angle $\alpha(\phi', \phi)$ by direct computation for the transformations $\bar{g} = \bar{g}(b_a, \beta)$ the results

$$\begin{aligned} \text{for } \bar{g}(0, 0) &: \alpha(\phi, \phi) = 0, \\ \text{for } \bar{g}(0, \beta) &: \frac{e}{\hbar c}\alpha(\phi', \phi) = -\tilde{g}'\beta, \\ \text{for } \bar{g}(b_a, 0) &: \frac{e}{\hbar c}\alpha(\phi', \phi) = 0. \end{aligned} \quad (\text{A16})$$

The last line in (A16) implies that there are no residual $SU(2)_W$ gauge transformations left on the stability subgroup $H = U(1)_{e.m.}$. After transforming to the origin $\hat{\phi}$ in \tilde{G}/H only one gauge degree of freedom remains which is of the form (A15) with $\alpha(\phi', \phi)$ given by (A16) together with the corresponding transformation rule for the electromagnetic potentials A_μ [see Appendix B]. The result for the hypercharge transformation $\bar{g}(0, \beta)$ quoted in (A16) follows also directly from the determinant of the transformation (A14).

APPENDIX B: THE ELECTROMAGNETIC GAUGE

We call the gauge obtained by realizing the transformations $U(\bar{g}(b_a, \beta))$ of \tilde{G} in terms of transformations $U(h(\phi', \phi))$ of the electromagnetic subgroup $H = U(1)_+$ of \tilde{G} the electromagnetic or nonlinear gauge [compare Eqs. (A14) and (A15)]. To characterize this gauge, which we shall denote by a hat, the scalar field ϕ of the theory is used: As described in Appendix A, ϕ takes the form $\hat{\phi} = \begin{pmatrix} 0 \\ \hat{\varphi}_0 \end{pmatrix}$ in this gauge, and the residual gauge transformations are the transformations of the stability subgroup $U(1)_+ = U(1)_{e.m.}$ leaving $\hat{\phi}$ invariant.

It is of particular interest to determine the form the gauge potentials A_μ^a and B_μ take in the electromagnetic gauge, i.e. determine \hat{A}_μ^a and \hat{B}_μ and their residual gauge freedom given by the transformations $h(\phi', \phi)$. Let us to this end rewrite the G -covariant derivative (2.28) of ϕ in terms of the spherical components $A_\mu^- = W_\mu$, $A_\mu^+ = W_\mu^\dagger$ and A_μ^3 [compare

(2.75)] and express A_μ^3 and B_μ in terms of A_μ and Z_μ with the help of (2.73). One finds with $\tau_\pm = \frac{1}{2\sqrt{2}}(\tau_1 \pm i\tau_2)$:

$$\tilde{\tilde{D}}_\mu \phi = \left[D_\mu + i\tilde{g} (W_\mu \tau_+ + W_\mu^\dagger \tau_-) + i\tilde{e} \frac{e}{\hbar c} \hat{q} A_\mu + i\tilde{g}_0 \left(\frac{1}{2} \tau_3 - \sin^2 \theta_W \cdot \hat{q} \right) Z_\mu \right] \phi. \quad (\text{B1})$$

Performing now a gauge transformation with $U^{-1}(\phi)$, mapping ϕ into $\hat{\phi}$, the covariant derivative (B1) of ϕ is mapped into

$$\hat{\tilde{\tilde{D}}}_\mu \hat{\phi} = \left[D_\mu + i\tilde{g} \hat{W}_\mu \tau_+ - \frac{i}{2} \tilde{g}_0 \hat{Z}_\mu \right] \hat{\phi}, \quad (\text{B2})$$

where we have used the fact that $\hat{q}\hat{\phi} = 0$, $\tau_- \hat{\phi} = 0$ and $\frac{1}{2} \tau_3 \hat{\phi} = -\frac{1}{2} \hat{\phi}$. We rewrite this for later use in terms of isospinor components, i.e.

$$\hat{\tilde{\tilde{D}}}_\mu \hat{\phi} = \begin{pmatrix} 0 \\ D_\mu \hat{\varphi}_0 \end{pmatrix} + i \begin{pmatrix} \frac{1}{\sqrt{2}} \tilde{g} \hat{\varphi}_0 \hat{W}_\mu \\ -\frac{1}{2} \tilde{g}_0 \hat{\varphi}_0 \hat{Z}_\mu \end{pmatrix}, \quad (\text{B3})$$

where $D_\mu \hat{\varphi}_0 = \partial_\mu \hat{\varphi}_0 + \frac{1}{2} \kappa_\mu \hat{\varphi}_0$ is the Weyl-covariant derivative of the real field $\hat{\varphi}_0$. From the residual transformation with $U(h(\phi', \phi))$ given by (A15) one now concludes that

$$\left(\hat{\tilde{\tilde{D}}}_\mu \hat{\phi} \right)' = \begin{pmatrix} e^{-i\frac{e}{\hbar c} \alpha(\phi', \phi)} & 0 \\ 0 & 1 \end{pmatrix} \hat{\tilde{\tilde{D}}}_\mu \hat{\phi} \quad (\text{B4})$$

is the nonlinear subgroup transformation rule for the G -covariant derivative of $\hat{\phi}$. One sees at once from the split form (B3) that (B4) implies for the residual (electromagnetic) $U(1)_+$ gauge transformations of the potentials \hat{Z}_μ , \hat{W}_μ and \hat{W}_μ^\dagger the behaviour

$$\hat{Z}'_\mu = \hat{Z}_\mu, \quad (\text{B5})$$

$$\hat{W}'_\mu = e^{-i\frac{e}{\hbar c} \alpha(\phi', \phi)} \hat{W}_\mu, \quad (\text{B6})$$

$$\hat{W}'_\mu{}^\dagger = e^{+i\frac{e}{\hbar c} \alpha(\phi', \phi)} \hat{W}_\mu^\dagger. \quad (\text{B7})$$

For the electromagnetic potentials \hat{A}_μ applies the usual rule given by [see Eq. (B14) below]

$$\hat{A}'_\mu = \hat{A}_\mu + \partial_\mu \alpha(\phi', \phi). \quad (\text{B8})$$

It is clear from the definitions that \hat{A}_μ and \hat{Z}_μ are real vector fields.

In concluding this appendix we observe with respect to the theorem quoted at the end of Sect. II that the connection in the present case does *not* reduce from a *Lie* \tilde{G} -valued to a *Lie* H -valued form. In fact, we have for the *Lie* \tilde{G} -valued gauge potentials with Eqs. (2.73), (2.75) and (2.77):

$$\begin{aligned} \tilde{\Gamma}_\mu &= \tilde{g} \frac{1}{2} \tau_a A_\mu^a + \tilde{g}' \frac{1}{2} \mathbf{1} B_\mu \\ &= \begin{pmatrix} \tilde{e} \bar{A}_\mu + \tilde{g}_0 \left[\frac{1}{2} - \sin^2 \theta_W \right] Z_\mu & \tilde{g} \frac{1}{\sqrt{2}} W_\mu \\ \tilde{g} \frac{1}{\sqrt{2}} W_\mu^\dagger & -\frac{1}{2} \tilde{g}_0 Z_\mu \end{pmatrix} \end{aligned} \quad (\text{B9})$$

with $\bar{A}_\mu = \frac{e}{\hbar c} A_\mu$. In this notation the covariant derivative is written $\tilde{D}_\mu \phi = D_\mu \phi + i \tilde{\Gamma}_\mu \phi$. The transformation to the electromagnetic gauge with $U^{-1}(\phi)$ yields

$$\hat{\tilde{\Gamma}}_\mu = U^{-1}(\phi) \tilde{\Gamma}_\mu U(\phi) - i U^{-1}(\phi) \partial_\mu U(\phi) \quad (\text{B10})$$

which is still *Lie* \tilde{G} -valued and hence, according to the theorem of Sect. II, the \tilde{G} gauge symmetry does not reduce but is nonlinearly realized.

From (B10) we compute for the fields \hat{W}_μ and \hat{Z}_μ appearing in (B2) the expressions:

$$\begin{aligned} \frac{\tilde{g}}{\sqrt{2}} \hat{W}_\mu = & \left[\left(\tilde{e} \bar{A}_\mu + \tilde{g}_0 \left[\frac{1}{2} - \sin^2 \theta_W \right] Z_\mu \right) \varphi_0 \varphi_+ + \frac{\tilde{g}}{\sqrt{2}} W_\mu (\varphi_0)^2 - \frac{\tilde{g}}{\sqrt{2}} W_\mu^\dagger (\varphi_+)^2 \right. \\ & \left. + \frac{1}{2} \tilde{g}_0 Z_\mu \varphi_0 \varphi_+ \right] (\hat{\varphi}_0)^{-2} - i \left[\varphi_0 \partial_\mu \left(\frac{\varphi_+}{\hat{\varphi}_0} \right) - \varphi_+ \partial_\mu \left(\frac{\varphi_0}{\hat{\varphi}_0} \right) \right] (\hat{\varphi}_0)^{-1}, \end{aligned} \quad (\text{B11})$$

$$\begin{aligned} -\frac{1}{2} \tilde{g}_0 \hat{Z}_\mu = & \left[\left(\tilde{e} \bar{A}_\mu + \tilde{g}_0 \left[\frac{1}{2} - \sin^2 \theta_W \right] Z_\mu \right) |\varphi_+|^2 + \frac{\tilde{g}}{\sqrt{2}} W_\mu \varphi_+^* \varphi_0 + \frac{\tilde{g}}{\sqrt{2}} W_\mu^\dagger \varphi_0^* \varphi_+ \right. \\ & \left. - \frac{1}{2} \tilde{g}_0 Z_\mu |\varphi_0|^2 \right] (\hat{\varphi}_0)^{-2} - i \left[\varphi_+^* \partial_\mu \left(\frac{\varphi_+}{\hat{\varphi}_0} \right) + \varphi_0^* \partial_\mu \left(\frac{\varphi_0}{\hat{\varphi}_0} \right) \right] (\hat{\varphi}_0)^{-1}. \end{aligned} \quad (\text{B12})$$

For $\hat{\tilde{A}}_\mu = \frac{e}{\hbar c} \hat{A}_\mu$, by inserting also \hat{Z}_μ from (B12), one finds the relation:

$$\begin{aligned} \tilde{e} \hat{\tilde{A}}_\mu + \tilde{g}_0 \left[\frac{1}{2} - \sin^2 \theta_W \right] \hat{Z}_\mu = & \left[\left(\tilde{e} \bar{A}_\mu + \tilde{g}_0 \left[\frac{1}{2} - \sin^2 \theta_W \right] Z_\mu \right) |\varphi_0|^2 - \frac{\tilde{g}}{\sqrt{2}} W_\mu \varphi_0 \varphi_+^* \right. \\ & \left. - \frac{\tilde{g}}{\sqrt{2}} W_\mu^\dagger \varphi_0^* \varphi_+ - \frac{1}{2} \tilde{g}_0 Z_\mu |\varphi_+|^2 \right] (\hat{\varphi}_0)^{-2} - i \left[\varphi_0 \partial_\mu \left(\frac{\varphi_0^*}{\hat{\varphi}_0} \right) + \varphi_+ \partial_\mu \left(\frac{\varphi_+^*}{\hat{\varphi}_0} \right) \right] (\hat{\varphi}_0)^{-1}. \end{aligned} \quad (\text{B13})$$

After the transformation (B10) has been carried out the residual gauge freedom is given by the transformations $U(h(\phi', \phi))$ of the electromagnetic subgroup $U(1)_+ = U(1)_{e.m.}$, i.e. by

$$\hat{\tilde{\Gamma}}_\mu' = U(h(\phi', \phi)) \hat{\tilde{\Gamma}}_\mu U^{-1}(h(\phi', \phi)) - i U(h(\phi', \phi)) \partial_\mu U^{-1}(h(\phi', \phi)), \quad (\text{B14})$$

with $U(h(\phi', \phi))$ as defined in (A15) with phase angle (A16). For the potentials $\hat{A}'_\mu, \hat{Z}'_\mu, \hat{W}'_\mu$ and \hat{W}'_μ^\dagger this yields at once the relations (B5) – (B8).

We finally quote the form of the fermion fields after transformation to the origin in \tilde{G}/H :

$$\hat{\psi}_L = U^\dagger(\phi) \psi_L \quad \text{and} \quad \hat{\psi}_R = \psi_R, \quad (\text{B15})$$

implying that

$$\hat{\nu}_L = (\hat{\varphi}_0)^{-1} [\varphi_0 \nu_L - \varphi_+ e_L], \quad \hat{e}_L = (\hat{\varphi}_0)^{-1} [\varphi_+^* \nu_L + \varphi_0^* e_L], \quad \hat{e}_R = e_R. \quad (\text{B16})$$

The residual $U(1)_{e.m.}$ gauge transformations of these fields are

$$\hat{\nu}_L' = e^{-i \frac{e}{\hbar c} \alpha(\phi', \phi)} \hat{\nu}_L, \quad \hat{e}_L' = \hat{e}_L, \quad \hat{e}_R' = \hat{e}_R, \quad (\text{B17})$$

and correspondingly for the adjoint fields $\hat{\nu}_L, \hat{e}_L$ and \hat{e}_R . The Yukawa coupling (2.27) of ϕ and the fermion fields may in this notation, together with $U^\dagger(\phi) \phi = \hat{\phi}$, be written as

$$\tilde{\gamma} \left\{ (\bar{\psi}_L \phi) \psi_R + \bar{\psi}_R (\phi^\dagger \psi_L) \right\} \equiv \tilde{\gamma} \left\{ (\hat{\psi}_L \hat{\phi}) \hat{\psi}_R + \hat{\psi}_R (\hat{\phi}^\dagger \hat{\psi}_L) \right\} = \tilde{\gamma} \hat{\phi}_0 \left\{ \hat{e}_L \hat{e}_R + \hat{e}_R \hat{e}_L \right\}. \quad (\text{B18})$$

The r.-h. side of (B18) may be written as $\tilde{\gamma} \hat{\phi}_0 \hat{\tilde{\psi}}_e \hat{\psi}_e$, with $\hat{\psi}_e$ denoting the electron field transformed to the origin in \tilde{G}/H , showing that the Yukawa coupling represents, in effect, an electron mass term.

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